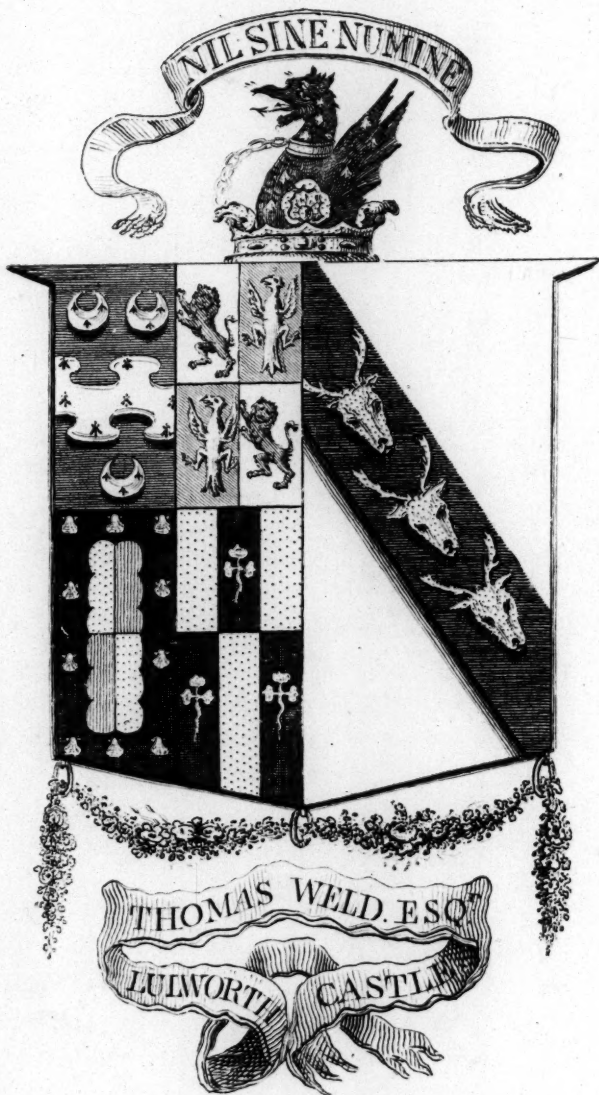
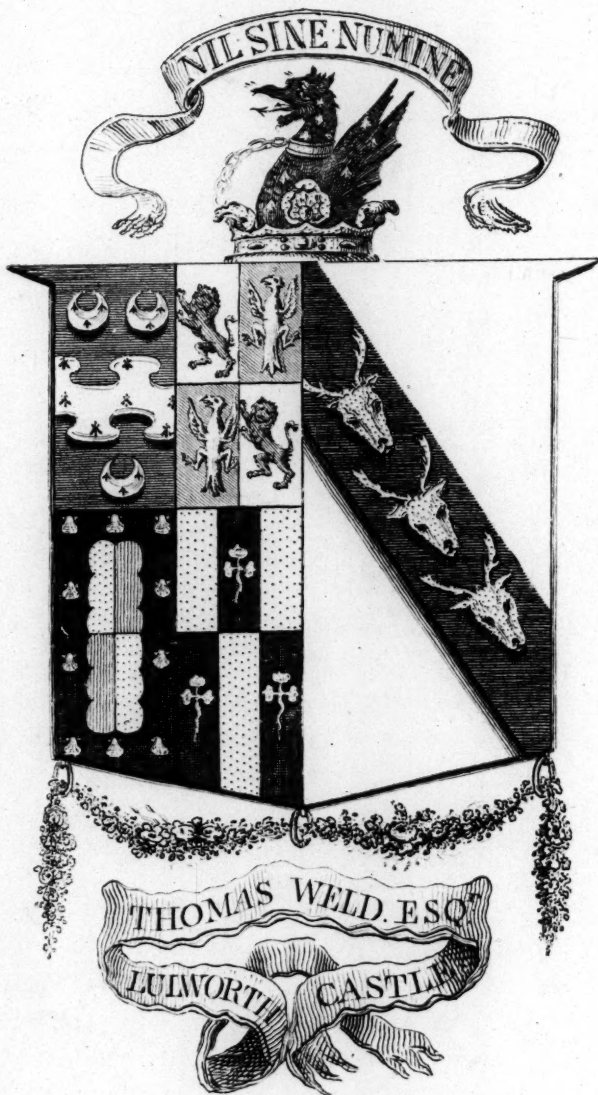
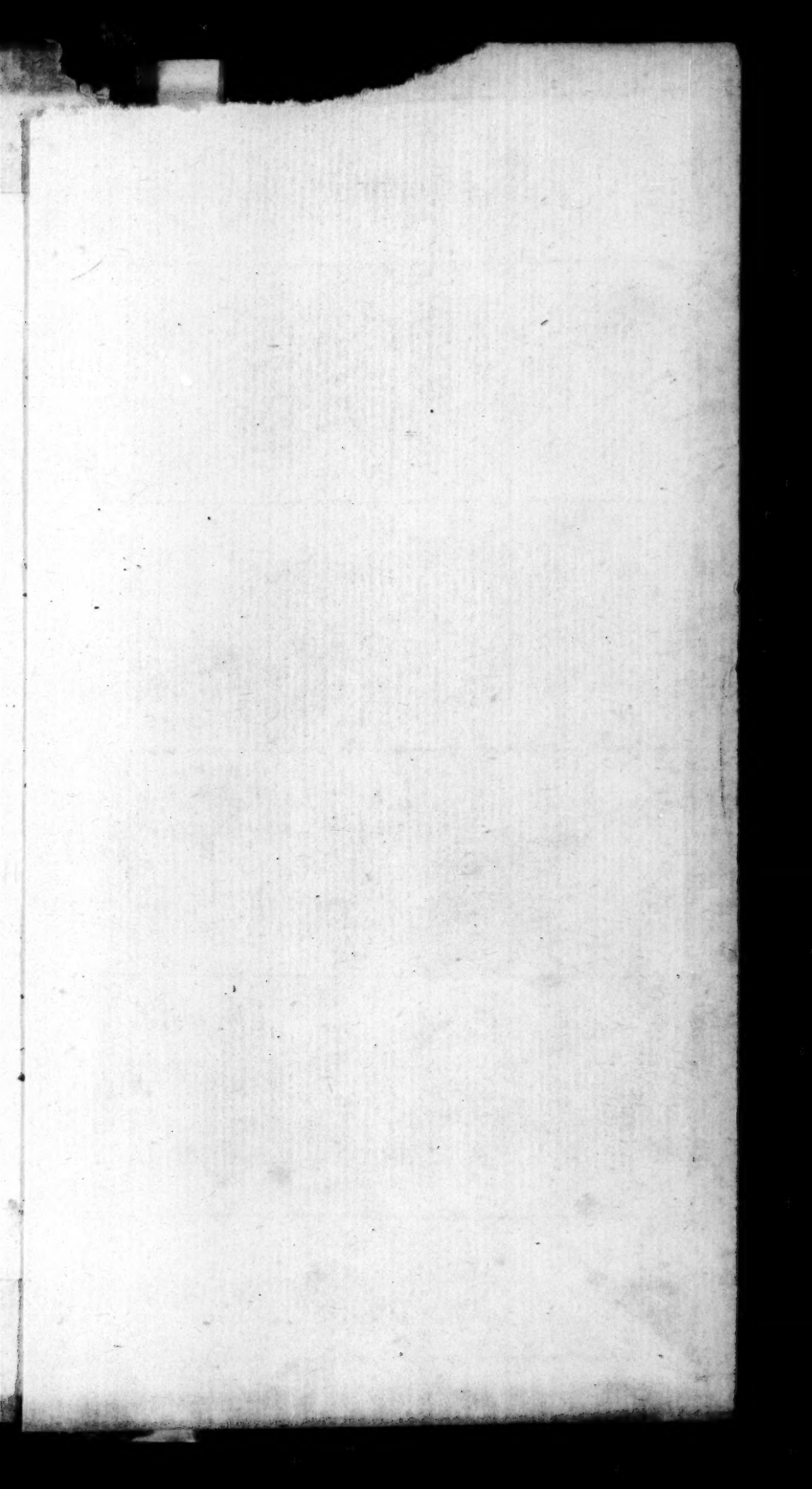


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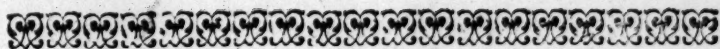


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And also the *Multiplication of Feet and Inches,*
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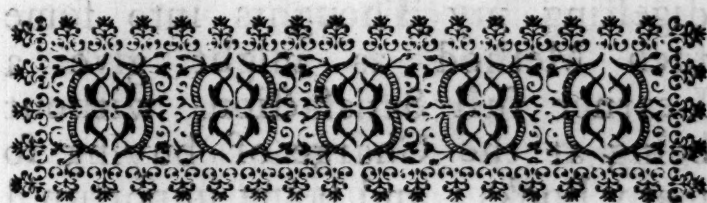
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E. Symon. 1717.



THE PREFACE.

HAVING perus'd several Books concerning the Mensuration of Superficies and Solids, and the Works of Artificers relating to Building ; but not finding any one Book so perfect as to give any tolerable Satisfaction to a Learner ; and I having practis'd and taught Measuring for several Years, and thereby gain'd Experience and Knowledge in that Art, having learn'd some Things from one Author, and some Things

A 3

from

The PREFACE.

from another, I began to think of digesting my Thoughts into some such Method as might give a Learner full Satisfaction, without being at the Charge of buying so many Books ; and being importun'd thereunto by some Friends, I fell to work, and at last brought them to that Perfection you here find the following Work.

1. As to the *Decimal Arithmetick*, I have been as brief as the Matter would well bear, to make it plain.

2. As to the multiplying of Feet and Inches, commonly call'd *Cross-Multiplication* ; my Method differs from that which is usually taught in other Authors, as being (I think) much shorter and plainer.

3. In measuring of Superficies and Solids, I have given the Demonstration of the Rules, which I thought might be very acceptable to

The PREFACE.

to the Ingenious ; for indeed I always look upon the writing of a Rule without a Demonstration, (in any Part of the Mathematicks) to be but lame and defective ; and for want of knowing the Reason of the Rule, a Learner may commit great Errors ; besides, when a Learner knows the Reason of the Rules, he may retain them better in his Memory. The Rule for measuring a Prismoid and Cylindroid, I had out of Mr. *Everard's Art of Gauging* ; but the Reason he does not shew, neither have I found it in any other Author ; but that the Method is true, I have endeavour'd to make plain.

The Demonstrations of the Rules for finding the Area of an Ellipsis and Parabola ; also the Demonstration of the Rules for finding the solid Content of the Frustum of a Cone and Pyramid, the Solidity of a Globe, of a Spheroid, a Parabolick Conoid, and of a Parabolick

The PREFACE.

rabolick Spindle, and their Frustums, I had from the ingenious Mr. *Ward's Young Mathematician's Guide*; where the curious and ingenious Reader may see many other Demonstrations algebraically perform'd: I have also demonstrated the Rule for finding the Solidity of a Globe out of *Par- die's Elements of Geometry*, (Book the 5th, Art. the 33d) publish'd in *English*, with many Additions, by the Reverend Dr. *Harris, F. R. S.* and the same is also done out of *Sturmius's Mathesis Enucleata*; so that the ingenious Reader may use which of those Ways he likes best.

The Scale suppos'd to be us'd in all the Operations, is the Line of Numbers, commonly call'd *Gunter's Line*, which is upon the ordinary two Feet, or eighteen Inch Rules, commonly us'd by Carpenters, Masons, &c. because I thought it needless, as well as impertinent,
to

The PREFACE.

to write the Use of Sliding-Rules, or any other particular Scales, they being sufficiently treated of by several Authors, *viz.* by the above-nam'd Mr. *Everard*, in his *Art of Gauging* above-mention'd, where you have the Use of a Sliding-Rule in Arithmetick, Geometry, in measuring of Superficies and Solids, Gauging, &c. Likewise Mr. *Hunt* has wrote largely of the Use of his Sliding-Rule, in Arithmetick, Geometry, Trigonometry, Gauging, Dyalling, &c. There are several others who have explain'd the Use of their own Rules; so that the more curious Readers may find full Satisfaction in those Authors.

One Thing I have omitted in the Book, which I think may not be very improperly inserted in this Place; that is, how to find a Number upon the Line. If the Number you would find, consists only of Units, then the Figures upon
upon

THE PREFACE.

upon the Line represent the Number sought: Thus, if the Number be 1, 2, 3, &c. then 1, 2, 3, &c. upon the Line, represents the Number sought. But if the Number consists of two Figures, that is, of Units and Tens, then the Figure upon the Rule stands for the Tens, and the larger Divisions stand for Units; thus, if 34 were to be found upon the Line, the Figure 3 upon the Line is 30, and 4 of the large Divisions (counted forward) is the Point representing 34; and if 340 were to be found, it will be at the same Point upon the Line; and if 304 were to be found, then the 3 upon the Line is 300, and 4 of the smaller Divisions (counted forward) is the Point representing 304. If the Number consists of 4 Places, or Thousands, then the Figure upon the Line stands for Thousands, and the larger Divisions are Hundreds, the lesser Divisions are Tens, and the tenth Parts of those
lesser

The P R E F A C E.

lesser Divisions are Units. Thus, if 2735 were to be found, then the 2 is 2000; and the 7 larger Divisions (counted forward) is 700 more; and 3 of the lesser Divisions is 30 more; and half of one of the lesser Divisions is 5 more, which is the Point representing 2735. You must remember, that between each Figure upon the Line, there are 10 Parts, which I call the larger Divisions; and each of those larger Divisions are subdivided (or suppos'd so to be) into 10 other Parts, which I call the smaller Divisions; and each of those Parts suppos'd to be subdivided again into 10 other Parts, &c. You must also remember, that if 1, in the Middle of the Line, stands only for 1, then 1 at the upper End will be 10, and 1 at the lower End will only be $\frac{1}{10}$; but if 1 at the lower End signifies 1, then 1 in the Middle stands for 10, and 1 at the upper End is 100, &c.

There

The P R E F A C E.

There is one Thing more which I would have my Reader to understand, and that is, How to find all such proportional Numbers made Use of in the Proportions about a Circle, and of a Cylinder, and in other Places; which Thing may be of good Use, to know how to correct a Number which may happen to be false printed, or to enlarge any Number to more decimal Places, for more Exactness; for though I have mention'd what such Numbers are, yet I have not shewn how to find them, which a Learner may be a little at a Nonplus to do; though they are easily found by the Rules there laid down. I shall therefore give two or three Examples, in this Place, of finding such Numbers, which may enable my Reader to find out the rest.

And, first, let it be requir'd to find the Area of a Circle, whose Diameter is an Unit.

By

The PREFACE.

By the Proportion of *Van Culen*, if the Diameter be 1, the Circumference will be 3.1415926, &c. whereof 3.1416 is sufficient in most Cases. Then the Rule teaches to multiply half the Circumference by half the Diameter, and the Product is the Area: That is, multiply 1.5708 by .5, (*viz.* half 3.1416 by half 1) and the Product is .7854, which is the Area of the Circle, whose Diameter is 1.

Again, if the Area be requir'd, when the Circumference is 1, first, find what the Diameter will be, thus, as 3.1416 : to 1, so is 1 to .318309, which is the Diameter when the Circumference is 1. Then multiply half .318309 by half 1; that is, .159154 by .5, and the Product is .079577, which is the Area of a Circle whose Circumference is 1.

The PREFACE.

If the Area be given, to find the Side of the Square equal, you need but extract the square Root of the Area given, and it is done: So the square Root of .7854 is .8862, which is the Side of a Square equal when the Diameter is 1. And if you extract the square Root of .079577, it will be .2821, which is the Side of the Square equal to the Circle, whose Circumference is 1.

If the Side of a Square within a Circle, be requir'd, if you square the Semidiameter, and double that Square, and out of that Sum extract the square Root, that shall be the Side of the Square which may be inscrib'd in that Circle; so, if the Diameter of the Circle be 1, then the half is .5; which squar'd, is .25, and this doubled is .5, whose square Root is .7071, the Side of the Square inscrib'd.

Again, if the Diameter of a Globe be 1, to find the Solidity. In Sect.
XI.

The P R E F A C E.

XI. Chap. II. it is demonstrated, that the Globe is $\frac{2}{3}$ of a Cylinder of the same Diameter and Altitude: Thus, if the Cylinder's Diameter be 1, and its Altitude or Length be also 1, find the Solidity thereof, and take $\frac{2}{3}$ of it, and that will be the Solidity of the Globe requir'd. Now, if the Diameter be 1, the Area of the Circle, or Base of the Cylinder, is .7854, (as is above shewn) which multiply'd by 1, the Altitude of the Cylinder, and the Product is also .7854, the Solidity of the Cylinder; $\frac{2}{3}$ whereof is .5236, which is the Solidity of the Globe, whose Diameter is 1.

From what has been said, the Reader may easily perceive how all other proportional Numbers are found, and may examine them at his Pleasure.

The PREFACE.

I shall not enlarge any farther upon the Matter, but leave the Book to speak for it self; and if it prove beneficial to the ingenious Practitioners, I have my Desire. So, wishing my ingenious Reader good Success in his Endeavours, not doubting but he will reap Profit hereby; which that he may, is the hearty Desire of his Well-wisher,

W. Hawney.

THE



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THE



THE Compleat Measurer, &c.

PART I.

CHAP. I.

Notation of DECIMALS.



DECIMAL Fraction is an artificial Way of setting down and expressing of Natural or Vulgar Fractions, as whole Numbers: And whereas the Denominators of Vulgar Fractions are divers, the Denominators of Decimal Fractions are always certain: For a Decimal Fraction hath always for its Denominator an Unit, with a Cypher or Cyphers annex'd to it, and must therefore be either 10, 100, 1000, 10000, &c. And therefore, in writing down of a Decimal Fraction, there is no Necessity of writing down the Denominator; for by bare Inspection it is certainly known, it consisting of an Unit, with as many Cyphers annex'd to it as there are Places (or Figures) in the Numerator.

B

Example,

Example. This Decimal Fraction $\frac{25}{100}$ may be written thus, .25, its Denominator being known to be an Unit with two Cyphers; because there are two Figures in the Numerator. In like Manner, $\frac{125}{1000}$ may be thus written, .125; and $\frac{3575}{10000}$ thus, .3575; and $\frac{7}{100}$ thus, .075; and $\frac{65}{100000}$ thus, .0065.

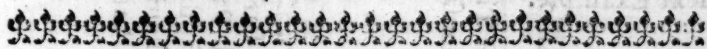
As whole Numbers increase in a Decuple, or ten-fold Proportion, towards the left Hand, so, on the contrary, Decimals decrease towards the right Hand, in a decuple Proportion, as in the following Scheme.

Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenth Parts.	Hundredth Parts.	Thousandth Parts.	Ten Thousandth Parts.	Hundred Thousandth Parts.	Milliounth Parts, &c.
7	6	5	4	3	2	1	0	1	2	3	4	5	6

Hence it appears, that Cyphers put on the right Hand of whole Numbers, do increase the Value of those Numbers in a decuple (or tenfold) Proportion; but being annex'd to the right Hand of a Decimal Fraction, do neither increase nor decrease the Value thereof: So $\frac{2500}{10000}$ is equivalent to $\frac{25}{100}$ or .25. And, on the contrary, tho' in whole Numbers Cyphers prefix'd before them, do neither increase nor diminish the Value; yet Cyphers before a Decimal Fraction, do diminish its Value in a decuple Proportion: For .25, if you prefix a Cypher before it, becomes $\frac{25}{1000}$, or .025; and .125 is $\frac{125}{100000}$, by prefixing two Cyphers before it, thus, .00125. And therefore, when you are to write a Decimal Fraction, whose Denominator hath more Cyphers than there are

are Figures in the Numerator, they must be supply'd by prefixing so many Cyphers before the Figures of your Numerator; as, suppose $\frac{12}{1000}$ were to be written down without its Denominator; here, because there are three Cyphers in the Denominator, and but two Figures in the Numerator, therefore prefix a Cypher before 12, and set it down thus, .012.

The Integers are separated from the Decimals several Ways, according to Mens Fancies; but the best and most usual Way, is by a Point or Period; and if there be no whole Number, then a Point before the Fraction is sufficient: Thus, if you were to write down $317\frac{317}{1000}$, it may be thus express'd, 317.217; and $59\frac{25}{1000}$ thus, 59.0025; and $\frac{75}{1000}$ thus, .0075, &c.



CHAP. II.

Reduction of DECIMALS.

IN *Reduction of Decimals* there are three Cases: 1st, To reduce a vulgar Fraction to a Decimal. 2dly, To find the Value of a Decimal in the known Parts of Coin, Weights, Measures, &c. 3dly, To reduce Coin, Weights, Measures, &c. to a Decimal. Of these in their Order.

I. To reduce a vulgar Fraction to a Decimal.

The RULE.

As the Denominator of the given, Fraction is to its Numerator, so is an Unit (with a competent Number of Cyphers annex'd) to the Decimal requir'd.

Therefore, if to the Numerator given, you annex a competent Number of Cyphers, and divide the Re-

4 *Reduction of DECIMALS.* Part I

sult by the Denominator, the Quotient is the Decimal equivalent to the vulgar Fraction given,

Example 1. Let $\frac{3}{4}$ be given, to be reduc'd to a Decimal of two Places, or having 100 for its Denominator.

To 3 (the Numerator given) annex two Cyphers, and it makes 300; which divide by the Denominator 4, and the Quotient is .75, the Decimal requir'd, and is equivalent to $\frac{3}{4}$ given.

Note, That so many Cyphers as you annex to the given Numerator, so many Places must be prick'd off in the Decimal found; and if it shall happen that there are not so many Places of Figures in the Quotient, the Deficiency must be supply'd by prefixing so many Cyphers before the Quotient-Figures, as in the next Example.

Example 2. Let $\frac{3}{373}$ be reduc'd to a Decimal having six Places.

To the Numerator annex six Cyphers, and divide by the Denominator, and the Quotient is 5235; but it was requir'd to have six Places, therefore you must prefix two Cyphers before it, and then it will be .005235, which is the Decimal requir'd, and is equivalent to $\frac{3}{373}$.

See the Work of these two Examples.

$\begin{array}{r} 4)3.00(.75 \\ \underline{28} \\ 20 \\ \underline{20} \\ .. \end{array}$	$\begin{array}{r} 373)3.000000(.5235 \\ \underline{} \\ 1350 \\ \underline{2040} \\ 3210 \\ \underline{} \\ 345 \end{array}$
---	--

In the second Example there remains 345, which Remainder is very insignificant, it being less than $\frac{1}{1000000}$ Part of an Unit, and therefore is rejected.

II. To

II. To find the Value of a Decimal in the known Parts of Money, Weight, Measures, &c.

The RULE.

Multiply the given Decimal by the Number of Parts in the next inferior Denomination, and from the Product prick off so many Places to the right Hand as there were Places in the Decimal given; and multiply those Figures prick'd off by the Number of Parts in the next inferior Denomination, and prick off so many Places as before, and so continue to do, 'till you have brought it to the lowest Denomination requir'd.

Example 1. Let .7565 of a Pound Sterling be given to be reduc'd to Shillings, Pence, and Farthings.

Multiply by 20, by 12, and 4, as the Rule directs, and always prick off four Places to the right Hand, and you will find it to make 15s. 1d. 2q. See the Work.

.7565

20

15.1300

12

d. —

1.5600

4

q. —

2.2400

A more compendious Way of finding the Value of the Decimal of a Pound Sterling.

Double the first Figure, (or Place of Primes) and it makes so many Shillings; and if the next Figure (or Place of Seconds) be 5, or more than 5, for the

B 3

5 add

6 *Reduction of DECIMALS.* Part I

5 add another Shilling to the former Shillings; then for every Unit in the second Place, count ten, and to that add the Figure in the third Place, and reckon that so many Farthings; but if they make above 13, abate one, and if it be above 38, abate 2, and add the remaining Farthings to the Shillings before found.

Example 1. Let .695 of a Pound be reduc'd to Shillings, Pence, and Farthings.

First, double your 6, and it makes 12 s.; then take 5 out of 9, and for that reckon another Shilling, and it makes 13 s. and the 4 remaining is 4 Tens, and the 5 makes 45, which being above 38, you must therefore cast away 2, and there rests 43 Farthings, which is 10 d. $\frac{3}{4}$. So the Answer is 13 s. 10 d. $\frac{3}{4}$.

	l.	s.	d.
So the Value of .725	=	14	6
And the Value of .878	=	17	6 $\frac{3}{4}$
And the Value of .417	=	8	4

And so of any other.

Let .59755 of a Pound Troy be reduc'd to Ounces, Penny-weights, and Grains.

Multiply by 12, by 20, and by 24, and always prick off five Places towards the right Hand, and you will find the Answer to be 7 oz. 3 pwt. 10 gr. *ferè*. See the Work.

.59755

12

7.17060

20

3.41200

24

164800

82400

9.88800

oz. pwt. gr.

Facit 7 3 9.888

Let

Part I. *Reduction of DECIMALS.* 7

Let .43569 of a Tun be reduc'd to Hundreds, Quarters, and Pounds.

Multiply by 20, by 4, and by 28, and the Answer will be 8 C. 2 qrs. 24 lb. fere.

$$\begin{array}{r}
 .43569 \\
 \times 20 \\
 \hline
 8.71380 \\
 \times 4 \\
 \hline
 2.85520 \\
 \times 28 \\
 \hline
 23.94560
 \end{array}$$

G. qrs. lb
Facit 8 2 23.9456

Let .9595 of a Foot be reduc'd to Inches and Quarters.

$$\begin{array}{r}
 .9595 \\
 \times 12 \\
 \hline
 11.5140 \\
 \times 4 \\
 \hline
 2.0560
 \end{array}$$

Facit 11 Inches, 2 Quarters.

III. *To reduce the known Parts of Money, Weight, Measure, &c. to a Decimal.*

The RULE.

To the Number of Parts of the lesser Denomination given, annex a competent Number of Cyphers, and divide by the Number of such Parts that are contain'd in the greater Denomination, to which the Decimal is to be brought; and the Quotient is the Decimal sought.

Example

8 *Reduction of DECIMALS.* Part I

Example 1. Let 6 d. be reduc'd to a Decimal of a Pound.

To 6 annex a competent Number of Cyphers, (suppose 3) and divide the Result by 240, (the Pence in a Pound) and the Quotient is the Decimal requir'd.

$$\begin{array}{r} 240)6.000(.025 \\ \underline{000} \\ 1200 \\ \underline{000} \\ 0000 \\ \underline{000} \\ 0000 \end{array} \quad \text{Facit .025}$$

Example 2. Let $3d. \frac{3}{4}$ be reduc'd to the Decimal of a Pound, having six Places.

In $3d. \frac{3}{4}$ there are fifteen Farthings; therefore to 15 annex six Cyphers, (because there are to be six Places in the Decimal requir'd) and divide by 960, (the Farthings in a Pound) and the Quotient is .015625.

96|0)15.00000|0(.015625

540
600
240
480

Example 3. Let $3\frac{1}{4}$ Inches be reduc'd to the Decimal of a Foot, consisting of four Places.

In $3\frac{1}{4}$ Inches there are 13 Quarters; therefore to 13 annex four Cyphers, and divide By 48, (the Quarters in a Foot) and the Quotient is .2708.

$$48) 13.0000(.2708$$

$$\begin{array}{r} 340 \\ 400 \\ \hline 16 \end{array}$$

Example

Part I. Addition of DECIMALS. 9

Example 4. Let 9 C. 1 qr. 16 lb be reduc'd to the Decimal of a Tun, having six Places.

C. qr. lb		
9 1 16		224 0'1052.00000 0(.469642
4		
37 qrs.		15600
28		21600
302	Facit .469642	14400
75		9600
1052 Pounds.		6400
		1920



CHAP. III.

Addition of DECIMALS.

ADDITION of Decimals is perform'd the same Way as Addition of whole Numbers, only you must observe to place your Numbers right, that is, Units under Units, Primes under Primes, Seconds under Seconds, &c.

Example. Let 317.25, 17.125, 275.5, 47.3579, and 12.75, be added together into one Sum.

317.25
17.125
275.5
47.3579
12.75
Sum 669.9829

This is so plain, that more Examples I think needless.

CHAP. IV.

Subtraction of DECIMALS.

SUBTRACTION of Decimals is perform'd likewise the same Way as in whole Numbers, Respect being had to the right placing the Numbers, (as in Addition) as in the following Examples.

(1)

From 212.0137
Subtr. 31.1275

Rem. 180.8862

Proof 212.0137

(2)

From 201.1250
Subtr. 5.5785

Refts 195.5465

Proof 201.1250

(3)

From 2051.315
Subtr. 79.172

Refts 1972.143

Proof 2051.315

(4)

From 30.5
Subtr. 7.2597

Refts 23.2403

Proof 30.5

Note, If the Number of Places in the Decimals be more in that which is to be subtracted, than in that which you subtract from, you must suppose Cyphers to make up the Number of Places, as in the fourth Example.

Part I. *Multiplication of DECIMALS.* II

CHAP. V.

Multiplication of DECIMALS.

MULTIPLICATION of Decimals is also perform'd the same Way as Multiplication of whole Numbers ; but to know the Value of the Product, observe this Rule.

Cut off, or separate by a Comma, or Prick, so many Decimal Places in the Product, as there are Places of Decimals in both Factors, viz. in the Multiplicand and Multiplier, which I shall farther explain in the following Examples.

Let .3125 be multiply'd by 2.75; multiply the Numbers together, as if they were whole Numbers, and the Product is 8.59375; and because there were three Places of Decimals prick'd off in the Multiplicand, and two Places in the Multiplier, therefore you must prick off five Places of Decimals in the Product, as you may see by the Work.

$$\begin{array}{r} 3125 \\ \times 275 \\ \hline \end{array}$$

$$\begin{array}{r} 15625 \\ + 21875 \\ + 6250 \\ \hline \end{array}$$

$$8.59375$$

Let

12 *Multiplication of DECIMALS.* Part I

Let 79.25 be multiply'd by .459.

In this Example, because two Places of Decimals are prick'd off in the Multiplicand, and three in the Multiplier, therefore there must be five prick'd off in the Product.

79.25

.459

71325

39625

31700

36.37575

Let .135272 be multiply'd by .00425.

In this Example, because in the Multiplicand are six Decimal Places, and in the Multiplier five Places; therefore in the Product there must be eleven Places of Decimals; but when the Multiplication is finish'd, the Product is but 57490600, viz. only eight Places; therefore, in this Case, you must prefix three Cyphers before the Product-Figures, to make up the Number of eleven Places; so the true Product will be .00057490600.

.135272

.00425

676360

279544

541088

.00057490600

Part I. Multiplication of DECIMALS. 13

More Examples for Practice.

$$\begin{array}{r}
 .001472 \\
 .1045 \\
 \hline
 7360 \\
 5888 \\
 1472 \\
 \hline
 .0001538240
 \end{array}$$

$$\begin{array}{r}
 .017532 \\
 347 \\
 \hline
 122724 \\
 70128 \\
 52596 \\
 \hline
 6.083604
 \end{array}$$

$$\begin{array}{r}
 279.25 \\
 .445 \\
 \hline
 139625 \\
 111700 \\
 111700 \\
 \hline
 124.26625
 \end{array}$$

$$\begin{array}{r}
 32.0752 \\
 .0325 \\
 \hline
 1603760 \\
 641504 \\
 962256 \\
 \hline
 1.04244400
 \end{array}$$

$$\begin{array}{r}
 4.443 \\
 15.98 \\
 \hline
 35544 \\
 39987 \\
 22215 \\
 4443 \\
 \hline
 70.99914
 \end{array}$$

$$\begin{array}{r}
 20.0291 \\
 35.45 \\
 \hline
 1001455 \\
 801164 \\
 1001455 \\
 600873 \\
 \hline
 710.031595
 \end{array}$$

$$\begin{array}{r}
 7.3564 \\
 .0126 \\
 \hline
 441384 \\
 147128 \\
 73564 \\
 \hline
 .09269064
 \end{array}$$

$$\begin{array}{r}
 .75432 \\
 .0356 \\
 \hline
 452592 \\
 377160 \\
 226296 \\
 \hline
 .026853792
 \end{array}$$

.09269064

C

.026853792

Con-

Contracted Multiplication of Decimals.

Because in Multiplication of Decimal Parts and mix'd Numbers, there is no Need to express all the Figures of the Product, but in most Cases two, three, or four Places of Decimals will be sufficient; therefore, to contract the Work, observe this following

R U L E.

Write the Unit's Place of the Multiplier under that Place of the Multiplicand whose Place you intend to keep in the Product; then invert the Order of all the other Figures, that is, write them all the contrary Way. Then, in multiplying, always begin at that Figure in the Multiplicand which stands over the Figure you are then multiplying withal, and set down the first Figure of each particular Product directly one under the other; but yet a due Regard must be had to the Increase arising from the Figures on the right Hand of that Figure in the Multiplicand which you begin to multiply at. This will appear more plain by Examples.

Example 1. Let 2.38645 be multiply'd by 8.2175, and let there be only four Places retain'd in the Decimals of the Product.

First, according to the Directions, write down the Multiplicand, and under it write the Multiplier, thus; place the 8 (being the Unit's Place of the Multiplier) under 4, the fourth Place of Decimals in the Multiplicand, and write the rest of the Figures quite contrary to the usual Way, as in the following Work: Then begin to multiply, first the 5 which is left out, (only with Regard to the Increase which must be carry'd from it) saying, 8 times 5 is 40, carry 4 in your Mind, and say 8 times 4 is 32, and 4 I carry is 36; set down 6 and carry 3, and proceed thro' the rest of the Figures, as in common Multiplication:

Then

Then begin to multiply with 2, saying, 2 times 4 is 8, for which I carry 1, (because it is above 5) and say 2 times 6 is 12, and 1 that I carry is 13; set down 3 and carry 1, and proceed thro' the rest of the Figures: Then multiply with one, saying, once 6 is 6, for which carry 1, and say once 8 is 8, and 1 is 9; set down 9, and proceed: Then multiply with 7, saying, 7 times 8 is 56, for which carry 6, (because it is above 55) and say 7 times 3 is 21, and 6 that I carry is 27; set down 7 and carry 2, and proceed: Then multiply with 5, saying, 5 times 3 is 15, for which carry 2, and say, 5 times 2 is 10, and 2 I carry is 12; which set down, and add all the Products together, and the total Product will be 19.6107. See the Work.

$$\begin{array}{r}
 2.38645 \\
 5712.8 \\
 \hline
 190916 \\
 4773 \\
 239 \\
 167 \\
 12 \\
 \hline
 19.6107
 \end{array}$$

Note, That in multiplying the Figure left out every Time next the right Hand in the Multiplicand, if the Product be 5, or upwards to 10, you carry 1, and if it be 15, or upwards to 20, carry 2; and if 25, or upwards to 30, carry 3, &c.

I have here set down the Work of the last Example, wrought by the common Way, by which you may see both the Reason and Excellency of this Way, all the Figures on the right Hand the Line being wholly omitted.

$$\begin{array}{r} 238645 \\ 82175 \\ \hline \end{array}$$

$$\begin{array}{r} 11 \overline{) 93225} \\ 167 \overline{) 0515} \\ 238 \overline{) 645} \\ 4772 \overline{) 90} \\ 190916 \overline{) 0} \end{array}$$

$$19.6106152875$$

Example 2. Let 375.13758 be multiply'd by 16.7324 , so that the Product may have but four Places of Decimals.

First, set 6, the Unit's Place of the Multiplier, under 5, being the fourth Place of Decimals in the Multiplicand, (because four Places of Decimals were to be prick'd off) and write all the rest of the Figures backward. Then multiply all the Figures of the Multiplicand by 1, after the common Way. Then begin with the second Figure of the Multiplier 6, saying 6 times 8 is 48, for which I carry 5, (in respect of the 8 left out) and 6 times 5 is 30, and 5 that I carry is 35; set down 5 and carry 3, and proceed after the common Method. Then begin with 7, the third Figure of the Multiplier, and say, 7 times 5 is 35, for which carry 4, and say, 7 times 7 is 49, and 4 I carry is 53; set down 3 under the first, and carry 5, and proceed as before. Then begin with 3, the fourth Figure of the Multiplier, and say, 3 times 7 is 21, carry 2, and say, 3 times 3 is 9, and 2 I carry is 11; set down 1, and carry 1, and proceed as before. Then begin with 2, the fifth Figure, and say, 2 times 3 is 6, for which I carry 1, and say, 2 times 1 is 2, and 1 I carry is 3; set down 3; and 2 times 5 is 10; set down 0, and carry 1, and proceed as before. Then begin with 4, the last Figure of the Multiplier, and say 4 times 1 is 4, for which I carry nothing, because 'tis less than 5; then.

then say, 4 times 5 is 20; set down 0, and carry 2, and proceed through the rest of the Figures of the Multiplicand. Then add all up together, and the Product is 6276.9520. See the Work.

375.13758 the Multiplicand.

4237.61 the Multiplier revers'd.

37513758 the Product with 1.

22508255 the Product with 6 increas'd with 6×8 .

2625963 the Product with 7 increas'd with 7×5 .

112541 the Product with 3 increas'd with 3×7 .

7503 the Product with 2 increas'd with 2×3 .

1500 the Product with 4 increas'd with 0.

6276.9520 the Product requir'd.

Let the same Example be repeated, and let only one Place in Decimals be prick'd off.

375.13758 the Multiplicand.

4237.61 the Multiplier inverted.

37514 the Product by 1 with the Increase of 1×7 .

22508 the Product with 6 increas'd with 6×3 .

2626 the Product with 7 increas'd with 7×1 .

113 the Product with 3 increas'd with 3×5 .

7 the Product with 2 increas'd with 2×7 .

1 the Increase only of 4×3 .

6276.9 the Product the same as before.

More Examples for Practice.

Multiply 395.3756 by .75642, and prick off four Places in Decimals.

395.3756 the Multiplicand.

24657. the Multiplier revers'd.

2767629 the Product by 7 increas'd with 7×6 .

197688 the Product by 5 increas'd with 5×5 .

23722 the Product by 6 increas'd with 6×6 .

1581 the Product by 4 increas'd with 4×3 .

79 the Product by 2 increas'd with 2×5 .

299.0699 the Product requir'd.

Let the same Example be repeated, and let there be only one Place of Decimals.

395.3756

24657.

2767 the Product by 7 increas'd with 7×3 .

198 the Product by 5 increas'd with 5×5 .

24 the Product by 6 increas'd with $6 \times 9 + 6 \times 5$.

2 the Increase of $4 \times 9 + 4 \times 3$.

299.1 the Product.

Characters

Characters, and their Signification.

Note, That this Mark $+$ signifies Addition; as $8+5$, that is, 8 more 5, or 8 added to 5; and $8+3+7$ denotes these Numbers are to be added into one Sum.

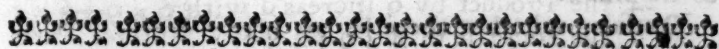
This Mark $-$ signifies Subtraction, as $9-4$ signifies that 4 is to be taken from 9:

This Mark $\times$ signifies Multiplication, as 7×5 signifies that 7 is to be multiply'd into 5.

This Mark $\div$ signifies Division, as $12 \div 4$ signifies 12 is to be divided by 4.

This Mark $=$ signifies Equality, or Equation, that is, when $=$ is plac'd between Numbers, or Quantities, it denotes them to be equal, as $7+5=12$, that is, 7 more 5 is equal to 12; and $15-7=8$, that is, 15 less by 7, is equal to 8, or subtract 7 from 15 and there remains 8.

This Mark $::$ is the Sign of Proportion, or the Golden Rule, it being always plac'd betwixt the two middle Terms or Numbers in Proportion, thus, $4:20::6:30$, to be thus read, as 4 is to 20, so is 6 to 30.



C H A P. VI.

Division of DECIMALS.

DIVISION of Decimals is perform'd after the same Manner as Division of whole Numbers; but to know the Value or Denomination of the Quotient, is the only Difficulty, for the resolving of which, observe either of the following

RULES.

R U L E S.

I. The first Figure in the Quotient must be of the same Denomination with that Figure in the Dividend which stands (or is suppos'd to stand) over the Unit's Place in the Divisor.

II. When the Work of Division is ended, count how many Places of decimal Parts there are in the Dividend more than in the Divisor; for that Excess is the Number of Places which must be separated in the Quotient for Decimals: But if there be not so many Figures in the Quotient, as is the said Excess, that Deficiency must be supply'd with Cyphers in the Quotient, prefix'd before the significant Figures thereof, towards the left Hand, with a Point before them; so shall you plainly discover the Value of the Quotient.

These following Directions ought also to be carefully observ'd.

If the Divisor consists of more Places than the Dividend, there must be a competent Number of Cyphers annex'd to the Dividend, to make it consist of as many (at least) or more Places of Decimals than the Divisor; for the Cyphers added must be reckon'd as Decimals.

Consider whether there be as many decimal Parts in the Dividend as there are in the Divisor; if there be not, make them so many, or more, by annexing of Cyphers.

In dividing of whole or mix'd Numbers, if there be a Remainder, you bring down more Cyphers, and, by continuing your Division, carry the Quotient to as many Places of Decimals as you please.

These Things being consider'd, I shall proceed to the Practice of Division of Decimals, which I shall endeavour to explain in as familiar and easy as Method as possible.

Example

Part I. Division of DECIMALS. 21

Example 1. Let 48 be divided by 144.

In this Example the Divisor 144, is greater than the Dividend 48; therefore, according to the Directions above, I annex a competent Number of Cyphers, (*viz.* four) with a Point between them, and divide according to the usual Way.

$$\begin{array}{r}
 144 \overline{) 48.0000(.3333} \\
 \underline{480} \\
 480 \\
 \underline{480} \\
 480
 \end{array}$$

But, first, in seeking how often 144 in 480, (the first three Figures of the Dividend) I find the Unit's Place of the Divisor to fall under the first Place of Decimals; therefore the first Figure in the Quotient is in the first Place of Decimals. Or by the second Rule, there being four Places of Decimals in the Dividend, and none in the Divisor; so the Excess of decimal Places in the Dividend, above that in the Divisor, is four; so that when the Division is ended, there must be four Places of Decimals in the Quotient. See the Work.

Example 2. Let 217.75 be divided by 65.

First, in seeking how oft 65 in 217, (the first three Figures of the Dividend) I find the Unit's Place of the Divisor to fall under the Unit's Place of the Dividend; therefore the first Figure in the Quotient will be Units, and all the rest Decimals. Or by the second Rule, there being two Places of Decimals in the Dividend, and no Decimals in the Divisor, therefore the Excess of decimal Places in the Dividend, above the Divisor, is two; so when the Division is ended, separate two Places in the Quotient, towards the right Hand, by a Point. See the Work.

In this Example, 3, the Unit's Place of the Divisor falls under 6, the Ten's Place of the Dividend; therefore (by the first Rule) the first Figure in the Quotient is Tens: Or, by the second Rule, the Excess of Decimal Places in the Dividend, above the Divisor, is three, there being five Places of Decimals in the Dividend, and but two in the Divisor; so there must be three Places of Decimals in the Quotient.

Example 3. Let 267.15975 be divided by 13.25 .

$$\begin{array}{r} 20.162 \\ 13.25 \overline{) 267.15975} \\ \underline{265} \\ 21 \\ \underline{212} \\ 7 \\ \underline{65} \\ 97 \\ \underline{91} \\ 67 \\ \underline{65} \\ 25 \\ \underline{25} \\ 0 \end{array}$$

2159

8347

3975

In this Example, 3, the Unit's Place of the Divisor falls under 6, the Ten's Place of the Dividend; therefore (by the first Rule) the first Figure in the Quotient is Tens: Or, by the second Rule, the Excess of Decimal Places in the Dividend, above the Divisor, is three, there being five Places of Decimals in the Dividend, and but two in the Divisor; so there must be three Places of Decimals in the Quotient.

Example 4. Let 15.675159 be divided by 375.89 .

$375.89 \overline{) 15.675159} (.0417$

63955

263669

546

In this Example, 5, the Unit's Place of the Divisor falls under 7, the second Place of Decimals in the Dividend; therefore (by the first Rule) the first Figure in the Quotient is in the second Place of Decimals; so that you must put a Cypher before the first Figure in the Quotient. And by the second Rule, the Excess of decimal Places in the Dividend, above the Number of decimal Places in the Divisor, is 4; for the decimal Places in the Dividend is 6, and the Number of Places in the Divisor but two; therefore there must be four Places of Decimals in the Quotient. But the Division being finish'd after the common Way, the Figures in the Quotient are but three, therefore you must prefix a Cypher before the significant Figures,

Example 5. Let 72.1564 be divided by .1347.

$$.1347 \overline{) 72.1564535.68}$$

$$\begin{array}{r}
 4806 \\
 7654 \\
 9190 \\
 11080 \\
 \hline
 394
 \end{array}$$

In this Example, the Divisor being a Decimal, the first Figure thereof falls under the Ten's Place in the Dividend; therefore the Units (if there had been any) should fall under the Hundred's Place in the Dividend, and so the first Figure in the Quotient is Hundreds. And, by the second Rule, there being four Places of Decimals in the Dividend, and as many in the Divisor, so the Excess is nothing; but in dividing I put two Cyphers to the Remainders, and continue the Division to two Places farther, so have I two Places of Decimals. See the Work.

Example

Example 6. Let 125 be divided by .0457.

$$\begin{array}{r}
 .0457 \overline{) 125.0000} (2.735 \\
 \underline{914} \\
 3360 \\
 \underline{3199} \\
 1610 \\
 \underline{1371} \\
 2390 \\
 \underline{2285} \\
 105
 \end{array}$$

In this Example, the Unit's Place of the Divisor (if there had been any) would fall under the Unit's Place of the Dividend; therefore the first Figure of the Quotient is Units. And, by the second Rule, there being seven Places of Decimals in the Dividend, and but four Places in the Divisor, so the Excess is three; therefore there must be three Places of Decimals in the Quotient.

I shall set down only the Work of some few Examples more, and so proceed to Contracted Division.

$$\begin{array}{r}
 .00456 \overline{) .000059791} (.00131 \\
 \underline{1419} \\
 511 \\
 \underline{55}
 \end{array}$$

Chap. VI. Division of DECIMALS. 25

Let 1 be divided by 282.

$$282 \overline{) 1.00000000 (0.0035461 \text{ - } \text{feré.}}$$

In Division of Decimals, when the Divisor hath many figures, and it is required to continue the Division, the Value of the Remainder be put small, the 00000000 will sometimes be large and tedious; but it will be excellently corrected by the following Method.

$$.325 \overline{) 1.4000000 (1.2307}$$

By the first Rule of the Chapter, (Page 20) find what is the Value of the figure in the Quotient, then by knowing the Divisor's Denominations you may have as many 00000000 as you please, by taking away of the last Hand figure of the Divisor, which is convenient for the purpose of the Division, and then put many 00000000 in the place of the Divisor, and then divide as usual, and you shall have the following Quotient.

$$.042 \overline{) 495.00000 (11785.71}$$

$$\begin{array}{r} 75 \\ 330 \\ 360 \\ 240 \\ 300 \\ 60 \\ 1 \\ 18 \end{array}$$

Division

D

Division of DECIMALS contracted.

IN Division of Decimals the common Way, when the Divisor hath many Figures, and it is requir'd to continue the Division till the Value of the Remainder be but small, the Operation will sometimes be large and tedious; but may be excellently contracted by the following Method.

The Rule.

By the first Rule of this Chapter, (pag. 20) find what is the Value of the first Figure in the Quotient; then by knowing the first Figure's Denomination, you may have as many or as few Places of Decimals as you please, by taking as many of the left Hand Figures of the Divisor as you think convenient for the first Divisor; and then take as many Figures of the Dividend as will answer them; and in dividing, omit one Figure of the Divisor at each following Operation. A few Examples will make it plain.

Example 1. Let 721.17562 be divided by 2.25742, and let there be three Places of Decimals in the Quotient.

2.25742)

74302
 29312
 43948
 672229
 74302
 29312
 43948
 672229

5501
209

By the first Rule of this Chapter, (pag. 20) find what is the Value of the Figure in the Quotient; then by knowing the ~~and~~ Figure's Denominations you may have as many ~~as~~ few Places of Decimals as you please, by taking ~~any~~ any of the left Hand Figures of the Divisor as you think convenient for the first Divisor; and then put as many Figures of the Dividend as will answer ~~that~~ in dividing omit the Figures of

In this Example, the Unit's Place of the Divisor falls under the Hundred's Place in the Dividend; and it is requir'd that three Places of Decimals be in the Quotient; so there must be six Places in all, that is, three Places of whole Numbers, and three Places of Decimals. Then, because I can have the Divisor in the first six Figures of the Dividend, I cut off the 62 with a Dash of the Pen, as useless; then I seek how oft the Divisor in the Dividend, and the Answer is three times; put 3 in the Quotient, and multiply and subtract as in common Division, and the Remainder is 43946. Then prick off the 3 in the Divisor, and seek how oft the remaining Figures may be had in 43946, the Remainder, which can be but once; put 1 in the Quotient, and multiply and subtract, and the next Remainder is 21372. Then prick off the four in the Divisor, and seek how often the remaining Figures may be had in 21372, which will be 9 times; put 9 in the Quotient, multiply thus, saying, 9

D-2

times

times 4 is 36, for which I carry 4 (in respect of the 4 last prick'd off) and 9 times 7 is 63, and 4 is 67; set down 7 and carry 6, and so proceed till the Division be finish'd, always respecting the Increase made from the Figures prick'd off. Observe the Work, which will better inform you than many Words.

$$2.25743 \overline{) 721.17562(319.467}$$

67	7229
439	466
225	743
213	7232
203	1687
1955	450
902	972
152	4780
135	4458
1703	220
158	0201
1230	19

I have set down the Work of this last Example at large, according to the common Way, that thereby the Learner may see the Reason of the Rule, all the Figures on the right Side the perpendicular Line being wholly omitted.

Example

Example 1. Let 51715916 be divided by 8.7586197 and let it be requir'd that four Places of Decimals be prick'd off in the Quotient.

8.7586197)51715916.155904377

43792077

792841

788774

40087

37034

5058

4379

674

813

61

61

61

61

61

61

61

61

61

61

61

61

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61

61

61

In this Example, I can't have 8, the first Figure in the Divisor, in 5, the first Figure of the Dividend; so that the Unit's Place of the Divisor falls under the Hundred's Place of the Dividend; so that there will be seven Figures in the Quotient, that is, three of whole Numbers, and four of Decimals; therefore there must be 7 Figures in the Divisor, (because the Number of Places in the Divisor and Quotient will be equal) and there must be eight Places in the Dividend; so that I cut off the Figure 5 with a Dash, as useless. Thus having proportion'd the Dividend to the Divisor, and both to the Number of Places or Figures desir'd in the Quotient, I proceed to divide as before, saying, how often 8 in 51, which will be 5 times; put 5 in the Quotient, and multiply and subtract, and

D 3

and the Remainder is 7922841. Then I prick off the first Figure in the Divisor, 5, and seek how often the remaining Figures of the Divisor in the aforesaid Remainder, which I find 9 times; put 9 in the Quotient, and multiply thereby, saying, 9 times 5 (the Figure prick'd off) is 45, for which I carry 5, and say, 9 times 1 is 9, and 5 I carry is 14; set down 4, and carry 1, and proceed to multiply the rest of the Figures, and subtract, and the Remainder will be 40087. Then prick off the Figure 1, and seek how often 87586 in the Remainder in 40087, the Answer will be 0; so put 0 in the Quotient, and prick off the Figure 6, and seek how often 8758 in 40087, which to be 4 Times; put 4 in the Quotient, and multiply, say, 4 times 6, (the Figure last prick'd off) is 24, for which I carry 2, and say, 4 times 8 is 32, and 2 I carry is 34; set down 4, and carry 3; multiply the rest of the Figures, and subtract as before, and so proceed after the same Manner, until all the Figures of the Divisor be prick'd off, to the last Figure. See the Work.

Example 3. Let 25.1367 be divided by 217.3543, and let there be five Places of Decimals in the Quotient.

In this Example, 7, the Unit's Place of the Divisor, falls under 1, the first Place of Decimals; therefore the first Figure of the Quotient is in the first Place of Decimals, so the Quotient will be all Decimals: Then, because the Quotient-Figures, and the Figures of the Divisor, will be of an equal Number, dash off the 43 in the Divisor, and the 7 in the Dividend, as useless, and divide as before.

217-35|43

2.756756)7414.76717(2689.67118

5513512

19012551

16540136

24720157

22014048

2666109

2481080

185029

165405

19624

19197

327

276

15

28

Although I have hitherto directed for pro-
portioning the Division, to be done
into the Quotient, whether of Decimals, or
pleas, yet there is no necessity for it; but
you may carry on your Division to what Degree you
please, and you may stop at any time, as in the
the Division, in order to extract the Work, as in the
following Examples, where it is not required to quick
off any the Counting Number of VII.
be done according to Division.

The Extraction of the SQUARE ROOT.

If a Square Number be given.

To find the Root thereof, that is, to find out
such a Number, as being multiplied into itself
the Product shall be equal to the Number given.
Operation is called, The Extraction of the Square Root.
wherein observe the following Directions.

10.34254

1st, You must point your given Number, that is, make a Point or Prick over the Unit's Place, another upon the Hundred's, and so upon every second Figure throughout.

2^{dly}, Then seek the greatest square Number in the first Point towards the left Hand, placing the square Number under the first Point, and the Root thereof in the Quotient, and subtract the said square Number from the first Point, and to the Remainder bring down the next Point, and call that the Resolvend.

3^{dly}, Then double the Quotient, and place it, for a Divisor, on the left Hand of the Resolvend; and seek how often the Divisor is contain'd in the Resolvend, (reserving always the Unit's Place) and put the Answer in the Quotient, and also on the right Hand Side of the Divisor; then multiply by the Figure last put in the Quotient, and subtract the Product from the Resolvend, (as in common Division) and bring down the next Point to the Remainder, (if there be any more) and proceed as before.

A TABLE of Squares and Cubes, and their Roots.

Root	1	2	3	4	5	6	7	8	9
Squa	1	4	9	16	25	36	49	64	81
Cub.	1	8	27	64	125	216	343	512	729

Example 1. Let 4489 be a Number given, and let the Square Root thereof be requir'd.

$$\begin{array}{r} 4489(67 \\ \underline{36} \end{array}$$

127889 Resolvend.

889 Product.

First,

Chap. VII. Extraction of the Square Root. 35

First, Point the given Number, as before directed; then (by the little Table aforegoing) seek the greatest Square Number in 44, (the first Point to the left Hand) which you will find to be 36, and 6 the Root; put 36 under 44, and 6 in the Quotient, and subtract 36 from 44, and there remains 8. Then to that 8 bring down the other Point 89, placing it on the right Hand, so it makes 889 for a Resolvend; then double the Quotient 6, and it makes 12, which place on the left Hand for a Divisor, and seek how often 12 in 88, (reserving the Unit's Place) the Answer is 7 times; which put in the Quotient, and also on the right Hand Side of the Divisor, and multiply 127 by 7, (as in common Division) and the Product is 889, which subtracted from the Resolvend, there remains nothing; so is your Work finish'd; and the Square Root of 4489 is 67; which Root, if you multiply by it self, that is, 67 by 67, the Product will be 4489, equal to the given Square Number, and proves the Work to be right.

Example 2. Let 106929 be a Number given, and let the Square Root thereof be requir'd.

1	0	6	9	2	9	3	2	7
10	40	80	40	80	40	10	40	80
10	40	80	40	80	40	10	40	80

62)169 Resolvend.

124 Product.

647)4529 Resolvend.

4529 Product.

First, Point your given Number, as before directed, putting a Point upon the Units, Hundreds, and Tens of Thousands; then seek what is the greatest square Number in 10, (the first Point) which by the little

Table

36 *Extraction of the Square Root.* Part I

Table you will find to be 9, and 3 the Root thereof; put 9 under 10, and 3 in the Quotient, then subtract 9 out of 10, and there remains 1; to which bring down 69, the next Point, and it makes 169 for the Resolvend; then double the Quotient 3, and it makes 6, which place on the left Hand of the Resolvend for a Divisor, and seek how oft 6 in 16; the Answer is twice; put 2 in the Quotient, and also on the right Hand of the Divisor, making it 62. Then multiply 62 by the 2 you put in the Quotient, and the Product is 124; which subtract from the Resolvend, and there remains 45; to which bring down 29, the next Point, and it makes 4529 for a new Resolvend. Then double the Quotient 32, and it makes 64, which place on the left Side the Resolvend for a Divisor, and seek how oft 64 in 452, which you will find 7 times; put 7 in the Quotient, and also on the right Hand of the Divisor, making it 647, which multiply'd by the 7 in the Quotient, it makes 4529, which subtracted from the Resolvend, there remains nothing. So 327 is the square Root of the given Number.

Example 3. Let 2268741 be a square Number given, the Root whereof is requir'd.

$$\begin{array}{r}
 2268741 \\
 1 \\
 \hline
 25 \overline{) 126} \\
 \underline{125} \\
 3006 \overline{) 18741} \\
 \underline{18036} \\
 30122 \overline{) 70300} \\
 \underline{60244} \\
 301243 \overline{) 1013600} \\
 \underline{903719} \\
 \text{Remains } 121871
 \end{array}$$

Having

Chap. VII. *Extraction of the Square Root.* 37

Having pointed the given Number, as before directed, seek what is the greatest Square Number in the first Point 2, which is once; put 1, the Square, under 2, and 1, the Root thereof, in the Quotient; subtract 1 from 2, and there remains 1; to which bring down the next Point, 26, and set on the right Hand, making it 126; double the 1 in the Quotient, makes 2; set 2 on the left Hand for a Divisor, and ask how often 2 in 12, which will be 5 times; put 5 in the Quotient, and also on the right Hand of the Divisor, making it 25; multiply (as in common Division) 25 by 5, and subtract the Product, 125 from 126, and there remains 1. Bring down the next Point, 87, and it makes 187 for a new Resolvend; and double the 15 in the Quotient, it makes 30 for a new Divisor. Then seek how often 30 in 18, which you can't have; so you must put 0 in the Quotient, and also on the right Hand of the Divisor, and bring down the next Point, and it makes 18741 for another new Resolvend. Then seek how often 300 in 1874, which will be 6 times; put 6 in the Quotient, and also on the right Hand of the Divisor, multiply and subtract, and the Remainder will be 705. Now, if you have a Mind to find the Value of the Remainder, you may annex Cyphers, by two at a Time, to the Remainders, and so prosecute the Work to what Number of Decimal Parts you please; thus, to 705 annex two Cyphers, and it will make 70500, and the Quotient, doubled, is 3012 for a Divisor: Then seek how often 3012 in 7050, (rejecting the Unit's Place) which will be twice; put 2 in the Quotient, and also on the right Hand of the Divisor, and multiply and subtract as before, and the Remainder will be 10256; to which annex two Cyphers, and proceed as before, and you will get a 3 in the Quotient next. So the square Root of the given Number is 1506.23, which being squar'd, or multiply'd, by it self, and the last Remainder added, will make the given Number, as follows.

$$\begin{array}{r}
 1506.23 \\
 1506.23 \\
 \hline
 451869 \\
 301246 \\
 903738 \\
 753115 \\
 150623 \\
 \hline
 2268728.8129 \\
 \text{The Remainder add} \quad \underline{121871} \\
 \hline
 \text{Proof } 2268741.0000
 \end{array}$$

Some more Examples for Practice.

Example 1. 7596796(2735.128 Root.

$$\begin{array}{r}
 4 \\
 \hline
 47)359 \\
 329 \\
 \hline
 545)3067 \\
 2725 \\
 \hline
 3506)34296 \\
 33036 \\
 \hline
 55122)126000 \\
 110244 \\
 \hline
 551242)1575600 \\
 1102484 \\
 \hline
 5512448)4736600 \\
 44099584 \\
 \hline
 3212016
 \end{array}$$

Example

Chap.VII. Extraction of the Square Root. 39

Example 2. 751417.5745 (866.84 Root.
64

$$\begin{array}{r}
 166)1114 \\
 \underline{996} \\
 1726)11817 \\
 \underline{10356} \\
 17328)146157 \\
 \underline{138624} \\
 273364)753345 \\
 \underline{693456} \\
 59889
 \end{array}$$

If the given Number be a mix'd Number, viz. consisting of a whole Number and a Decimal together, make the Number of decimal Places even, that is, 2, 4, 6, 8, &c. that so there may a Point fall upon the Unit's Place of the whole Numbers, as in this last Example, and in that following.

40 *Extraction of the Square Root.* Part I

Example 3. Let 656714.37512 be given, to find the Square Root.

$$\begin{array}{r}
 656714.375120(8103.79 \text{ Root.} \\
 64 \\
 \hline
 161) 167 \\
 161 \\
 \hline
 16203) 61437 \\
 48609 \\
 \hline
 162067) 1282851 \\
 1134469 \\
 \hline
 1620749) 14838220 \\
 14586741 \\
 \hline
 \text{Remains } 251479
 \end{array}$$

In this Example there are five Places of Decimals ; therefore put a Cypher to it, to make it even, that so there may a Point fall upon 4, the Unit's Place.

To find the Square Root of a Fraction.

If it be a decimal Fraction, the Work differs nothing from the Examples afore-going, only you must be mindful to point your given Number aright ; for (as was before directed) the Number of Places must always be made even, and then begin to point at the right-Hand, as in whole Numbers.

If it be a vulgar Fraction, it must be reduc'd to a Decimal, by the first Rule of the second Chapter.

I shall give an Example or two in each Case, and so conclude this Chapter.

Let

Chap. VII. Extraction of the Square Root. 41

Let .125 be a decimal Fraction given, whose square Root is requir'd; and let it be requir'd to have four Places of Decimals in the Root.

$$\begin{array}{r}
 .12500000(.3535 \\
 9 \\
 \hline
 65)350 \\
 325 \\
 \hline
 703)2500\ 0000 \\
 2109 \\
 \hline
 7065)39100 \\
 35325 \\
 \hline
 3775
 \end{array}$$

In this Example there must be five Cyphers annex'd, because two Places in the Square, make but one in the Root.

Let the square Root of .00715 be requir'd.

$$\begin{array}{r}
 .007150(.084 \\
 64 \\
 \hline
 164)750 \\
 656 \\
 \hline
 94
 \end{array}$$

In this a Cypher is added, to make the Places even.

42. *Extraction of the Square Root.* Part I

Let $\frac{7}{8}$ be a vulgar Fraction given, whose square Root is requir'd.

$$\begin{array}{r}
 8)7000 \qquad (.87500000(.9354 \\
 \underline{64} \qquad \qquad 81 \\
 60 \qquad 183)650 \\
 \underline{56} \qquad \qquad 549 \\
 40 \qquad 1865)10100 \\
 \underline{40} \qquad \qquad 9325 \\
 .. \qquad 18704)77500 \\
 \qquad \qquad 74816 \\
 \hline
 \qquad \qquad 2684
 \end{array}$$

Reduce this $\frac{7}{8}$ to a Decimal, it makes .875; to which annex Cyphers, and extract the square Root, as if it was a whole Number. So the Root is .9354.

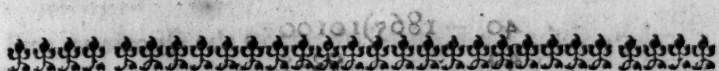
Let $\frac{3}{5}$ be a vulgar Fraction, whose square Root is requir'd.

$$\begin{array}{r}
 \dots\dots\dots \text{Root.} \\
 96)03.00000000|0 \qquad (.00312500(.0559 \\
 \dots\dots\dots 25 \\
 \underline{288} \qquad \qquad \qquad 105)625 \\
 \qquad \qquad \qquad \underline{525} \\
 120 \qquad \qquad \qquad 1109)10000 \\
 \underline{96} \qquad \qquad \qquad \underline{9981} \\
 240 \qquad \qquad \qquad 19 \\
 \underline{192} \\
 480 \\
 \underline{480}
 \end{array}$$

In

In extracting the Root of this, because the first Point consists of Cyphers, there must be a Cypher put first in the Quotient.

To prove this Rule, square the Root, and to the Product add the Remainder, as was before directed. To square a Number, is to multiply it by it self; and to cube it, is to multiply the Square of the Number by the Number it self.



C H A P. VIII.

The Extraction of the CUBE ROOT.

TO extract the Cube Root, is nothing else but to find such a Number as being first multiply'd into it self, and then into that Product, produceth the given Number; which to perform, observe these following Directions.

1st, You must point your given Number, beginning with the Unit's Place, and make a Point or Dor over every third Figure towards the left Hand.

2^{dly}, Seek the greatest Cube Number in the first Point, towards the left Hand, putting the Root thereof in the Quotient, and the said Cube Number under the first Point, and subtract it therefrom, and to the Remainder bring down the next Point, and call that the Resolvend.

3^{dly}, Triple the Quotient, and place it under the Resolvend; the Unit's Place of this under the Ten's Place of the Resolvend; and call this the triple Quotient.

4^{thly}, Square the Quotient, and triple the Square, and place it under the triple Quotient; the Units of this

this under the Ten's Place of the triple Quotient, and call this the triple Square.

5thly, Add these two together, in the same Order as they stand, and the Sum shall be the Divisor.

6thly, Seek how often the Divisor is contain'd in the Resolvend, rejecting the Unit's Place of the Resolvend, (as in the square Root) and put the Answer in the Quotient.

7thly, Cube the Figure last put in the Quotient, and put the Unit's Place thereof under the Unit's Place of the Resolvend.

8thly, Multiply the Square of the Figure last put in the Quotient, into the triple Quotient, and place the Product under the last, one Place more to the left Hand.

9thly, Multiply the triple Square by the Figure last put in the Quotient, and place it under the last, one Place more to the left Hand.

10thly, Add the three last Numbers together, in the same Order as they stand, and call that the Subtrahend.

Lastly, Subtract the Subtrahend from the Resolvend, and if there be another Point, bring it down to the Remainder, and call that a new Resolvend, and proceed in all respects as before.

Example

Chap.VIII. *Extraction of the Cube Root.* 45

Example 1. Let 314432 be a Cubick Number, whose Root is requir'd.

314432 (68 Root.
216

98432 Resolvend.

18 Triple Quotient of 6.
108 Triple Square of the Quotient 6.

1098 Divisor.

512 Cube of 8, the last Figure of the Root.
1152 The Square of 8, by the triple Quotient.
864 The triple Square of the Quotient 6 by 8.

98432 The Subtrahend.

.....

After you have pointed the given Number, seek what is the greatest Cube Number in 314, the first Point, which, by the former little Table, (Page 34) you will find to be 216, which is the nearest that is less than 314, and its Root is 6; which put in the Quotient, and 216 under 314, and subtract it therefrom, and there remains 98; to which bring down the next Point, 432, and annex to 98; so will it make 98432 for the Resolvend. Then triple the Quotient 6, it makes 18, which write down, the Unit's Place 8 under 3 the Ten's Place of the Resolvend. Then square the Quotient 6, and triple that Square, and it makes 108, which write under the triple Quotient, one Place to the left Hand; then add those two Numbers together, and they make 1098 for the Divisor. Then seek how often the Divisor is contain'd in the Resolvend, (rejecting the Unit's Place thereof) that is, how often 1098 in 9843, which is 8 times;

8 times; put 8 in the Quotient, and the Cube thereof below the Divisor, the Unit's Place under the Unit's Place of the Resolvend. Then square the 8 last put in the Quotient, and multiply 64, the Square thereof, by the triple Quotient, 18, the Product is 1152; set this under the Cube of 8, the Units of this under the Tens of that. Then multiply the triple Square of the Quotient by 8, the Figure last put in the Quotient, the Product is 864; set this down under the last Product, a Place more to the left Hand. Then draw a Line under those three, and add them together, and the Sum is 98432, which is call'd the Subtrahend; which being subtracted from the Resolvend, the Remainder is nothing; which shews the Number to be a true cubick Number, whose Root is 68; that is, if 68 be cub'd, it will make 314432.

For if 68 be multiply'd by 68, the Product will be 4624; and this Product, multiply'd again by 68, the last Product is 314432, which shews the Work to be right.

	68
	68

	544
The Work.	408

	4624
	68

	36992
	27744

The Proof	314432

Example 2. Let the Cube Root of 5735339 be requir'd.

After you have pointed the given Number, seek what is the greatest Cube Number in 5, the first Point, which

Chap. VIII. Extraction of the Cube Root. 47

which (by the little Table, pag. 34) you will find to be 1; which place under 5, and 1, the Root thereof, in the Quotient; and subtract 1 from 5, and there remains 4; to which bring down the next Point, it makes 4735 for the Resolvend. Then triple the 1, and it makes 3; and the Square of 1 is 1, and the Triple thereof is 3; which set one under another in their Order, which added makes 33 for the Divisor. Seek how often the Divisor in the Resolvend, and proceed as in the last Example.

5735339(179 Root.

4735 Resolvend.

3 The triple of the Quotient 1, the first Figure.

3 The triple Square of the Quotient 1.

33 The Divisor.

343 The Cube of 7, the second Figure of the Root.

147 The Square of 7, multip. in the triple Quotient 3.

21 The triple Square of the Quotient, multip. by 7.

3913 The Subtrahend.

822339 The new Resolvend.

51 The triple of the Quot. 17, the two first Fig.

867 The triple Square of the Quotient 17.

8721 Divisor.

729 The Cube of 9, the last Figure of the Root.

4131 The Square of 9, multip. by the trip. Quot. 51.

7803 The triple Square of the Quotient 867 by 9.

822339 The Subtrahend.

48 *Extraction of the Cube Root.* **Part I**

In this Example, 33, the first Divisor seems to be contain'd more than seven times in 4735, the Resolvend; but if you work with 9, or 8, you will find that the Subtrahend will be greater than the Resolvend.

Some more Examples for Practice.

32461759(319 The Root.

27

5461 Resolvend.

9 The triple of 3.

27 The triple Square of 3.

279 The Divisor.

1 The Cube of 1, the second Figure.

9 The triple Quotient, by the Square of 1.

27 The triple Square, multip. by 1, the 2d Figure.

2791 The Subtrahend.

2670759 A new Resolvend.

93 The triple of 31.

2883 The triple Square of 31.

28923 The Divisor.

729 The Cube of 9, the last Figure.

7533 The Square of 9, by 93, the triple Quotient.

25947 The triple Square 2883 by 9.

2670759 The Subtrahend.

.....

Chap.VIII. Extraction of the Cube Root. 49

84604519(439 The Root.
64

20604 Resolvend.

12 The triple of 4.
48 The triple Square of 4.

492 The Divisor.

27 The Cube of 3.
108 The Square of 3, by the triple Quotient.
144 The triple Square by 3.

15507 The Subtrahend.

5097519 The Resolvend.

129 The triple of 43.
5547 The triple Square of 43.

55599 The Divisor.

729 The Cube of 9.
10449 The Square of 9 by 129.
49923 The triple Square by 9.

5097519 The Subtrahend.

.....

259697989(638

216

43697 Resolvend.

18 The triple of 6.

108 The triple Square of 6.

1098 The Divisor.

27 The Cube of 3, the second Figure.

162 The Square of 3 by 18.

324 The triple Square 108 by 3.

34047 The Subtrahend.

9650989 Resolvend.

189 The triple of 63.

11907 The triple Square of 63.

119259 The Divisor.

512 The Cube of 8.

12096 The Square of 8 by 189.

95256 The triple Square 11907 by 8.

9647072 The Subtrahend.

3917 The Remainder.

Chap. VIII. Extraction of the Cube Root. 51

22069810125(2805
8

14069 Refolvend.

6 The triple of 2.

112 The triple Square of 2.

126 The Divisor.

512 The Cube of 8.

384 The Square of 8 by 6.

96 The triple Square by 8.

13552 The Subtrahend.

117810125 New Refolvend.

84 The triple of 28.

2352 The triple Square of 28.

23604 Divisor.

840 The triple of 280.

235200 The triple Square of 280.

2352840 New Divisor.

125 Cube of 5.

21000 Square of 5 by 840.

1176000 Triple Square by 5.

117810125 Subtrahend.

.....

In this Example 13952 being subtracted from the Refolv. 14069, the Remainder is 117, to which bring down 810, the third Point, and it makes 117810 for a new Refolv. and the next Divisor is 23604, which you cannot have in the said Refolvend; (the Unit's Place being rejected) so you must put 0 in the Quotient, and seek a new Divisor; (after you have brought down your last Point to the Refolvend) which new Divisor is 2352840; which you'll find to be contain'd 5 times. So proceed to finish the rest of the Work.

25917056(295.9
8

17917 The Resolvend.

6 The triple of 2.

12 The triple Square of 2.

126 The Divisor.

729 The Cube of 9, the 1d Figure.

486 The Square of 9 by 6.

108 The triple Square by 9.

16389 The Subtrahend.

1528056 The Resolvend.

87 The triple of 29.

2523 The triple Square of 29.

25317 The Divisor.

125 The Cube of 5, the 3d Fig.

2175 The Square of 5 by 87.

12615 The triple Square by 5.

1283375 The Subtrahend.

244681000 The Resolvend.

885 The triple of 295.

261075 The triple Square of 295.

2611635 The Divisor.

729 The Cube of 9, the last Figure.

71685 The Square of 9 by 885.

2349675 The triple Square by 9.

235685079 The Subtrahend.

8995921 The Remainder.

In this Example I annex 3 Cyphers to the Remainder, which makes the 3d Resolvend, by which Means I bring one Place of Decimals; and so you may proceed to more decimal Places at Pleasure, by annexing 3 Cyphers to the next Remainder, and carrying on the Work as before.

ICHA. VIII. Extraction of the Cube Root. 153.

93759.575070(45.42

64

29759 The Resolvend.

12

The triple of 4, the first Figure.

48

The triple Square of 4.

492

The Divisor.

125

The Cube of 5, the second Figure.

300

The Square of 5 by 12, the triple Quotient.

240

The triple Square by 5.

27125

The Subtrahend.

2634575

The Resolvend.

135

The triple of 45.

6075

The triple Square of 45.

60885

The Divisor.

64

The Cube of 4.

2160

The Square of 4 by 135.

24300

The triple Square by 4.

2451664

The Subtrahend.

182911070

The Resolvend.

1362

The triple of 45.4.

618348

The triple Square of 45.4.

6184842

The Divisor.

8

The Cube of 2.

5448

The Square of 2 by 1362.

1236696

The triple Square by 2.

123724088

The Subtrahend.

55186982

The Remainder.

In extracting the Cube Root of a mix'd Number, always observe to make the decimal Part to consist of either three, six, nine, &c. Places, that is, always to consist of even Points, as in the last Example, where the decimal Places were five, to which I annex'd a Cypher to make up six, and so I proceed to point it; and by that Means I have a Point falls upon the Unit's Place of whole Numbers, which you must always observe.

To extract the CUBE ROOT out of a Fraction.

This is the same to do as in whole Numbers, observe but the foregoing Directions for the true pointing thereof; for, as was before directed, the Decimal must always consist of either three, six, nine, &c. Places; and if it be not so, it must be made so, by annexing of Cyphers, as is abovesaid.

If the Cube Root of a vulgar Fraction be requir'd, you must first reduce it to a Decimal, and then extract the Root thereof.

Examples of each follow.

Example

Example 1. Let the Cube Root of .401719179 be requir'd.

.401719179 Root.

343

58719 Refolvend.

21 Triple of 7.

147 Triple Square of 7.

1491 Divisor.

27 Cube of 3.

189 Square of 3 by 21.

441 Triple Square by 3.

46017 Subtrahend.

12702179 Refolvend.

219 Triple of 73.

35987 Triple Square of 73.

160089 Divisor.

343 Cube of 7.

10731 Square of 7 by 219.

111909 Triple Square by 7.

11298553 Subtrahend.

1403626 Remainder.

Example

56 Extraction of the Cube Root II. Part II

Example 2. Let the Cube Root of .0001416 be requir'd.

.000141600(.052 Root.

125

16600 Resolvend.

15 The triple of 5.

75 Triple Square of 5.

765 Divisor.

8 Cube of 2.

60 The Square of 2 by 15.

150 Triple Square by 2.

15608 Subtrahend.

992 Remainder.

Example 3. Let $\frac{5}{27}$ be a vulgar Fraction, whose Cube Root is requir'd.

By the first Rule of Chapter II, reduce the vulgar Fraction to a Decimal.

276)5.000000000(.018115942

2240

320

440

1640

2600

1160

560

8

.0181159

0181159426 Root.

8

10115 Resolvend.

6 Triple of 2.

12 Triple Square of 2.

126 Divisor.

216 Cube of 6.

216 Square of 6 by the triple of 2.

72 The triple Square by 6.

9576 Subtrahend.

539942 Resolvend.

78 Triple of 26.

2028 Triple Square of 26.

20358 Divisor.

8 Cube of 2.

312 Square of 2 by 78.

4056 Triple Square 2028 by 2.

408728 Subtrahend.

131214 Remainder.

You may prove the Truth of the Work, by cubing the Root found, as was shew'd in the first Example; and if any Thing remain, add it to the said Cube, and the Sum will be the given Number, if the Work is rightly perform'd.

I will

58 *Multiplication of Feet, Inches, &c. Part I*

I will shew the Proof of the fifth Example, (*Page 50*) the given Number being 259697989, whose Root is 638, it being a surd Number, there remains 3917

$$\begin{array}{r}
 \text{---} 638 \\
 259697989 \\
 \text{---} \\
 5104 \\
 2914 \\
 \text{---} 3828 \\
 \text{---} \\
 \text{The Square } 407044 \\
 \text{---} 638 \\
 3256352 \\
 1221132 \\
 2442264 \\
 \text{---} \\
 \text{The Cube } 259694072 \\
 \text{The Remainder add } 3917 \\
 \text{---} \\
 \text{Proof equal to the given Number } 259697989
 \end{array}$$



C H A P. IX.

Multiplication of Feet, Inches, and Parts.

IN the multiplying of Feet, Inches, &c. I shall endeavour to lay down such easy and familiar Rules, as may be easily understood by the meanest Capacity.

Chap. IX. Multiplication of Feet, Inches, &c. 59

Example 1. Let 7 Feet, 9 Inches, be multiply'd by 3 Feet, 6 Inches.

F. I.

7 9
3 6

23 3 Prs.
3 10 6

27 1 6

First, Multiply 9 Inches by 3, saying, 3 times 9 is 27 Inches, which make 2 Feet 3 Inches; set down 3 under Inches, and carry 2 to the Feet, saying, 3 times 7 is 21, and 2 that I carry make 23; set down 23 under the Feet.

Then begin with 6 Inches, saying, 6 times 9 is 54 Parts, which is 4 Inches and 6 Parts; set down 6 Parts, and carry 4, saying, 6 times 7 is 42, and 4 that I carry is 46 Inches, which is 3 Feet 10 Inches; which set down, and add all up together, and the Product is 27 Feet 1 Inch 6 Parts.

Example 2. Let 75 Feet 7 Inches, be multiply'd by 9 Feet, 8 Inches.

F. I.

75 7
9 8

680 3
50 4 8

730 7 8

First, Multiply by 9 Feet, saying, 9 times 7 is 63, which is 5 Feet 3 Inches; set down 3 and carry 5, saying, 9 times 5 is 45, and 5 I carry is 50; set down 0, and carry 5, saying, 9 times 7 is 63, and 5

is

60 *Multiplication of Feet, Inches, &c. Part I*

is 68; set down 68, and proceed to multiply by 8 Inches, saying, 8 times 7 is 56, the Twelves in 56 is 4 times, and 8 remains; set 8 a Place to the right Hand, and carry 4: Then multiply 75 by 8, and the Product is 600, and 4 that I carry is 604, which divided by 12, the Quotient is 50 Feet, and 4 remains; set down 50 Feet 4 Inches, and add all up together, and you will find the Product 730 Feet, 7 Inches, 8 Parts.

I will repeat the last Example again, and shew another Way to work it, which, I think, is better, and more expeditious, when there are more Figures than one in the Feet; thus,

F. I.

75 7
9 8

680 3

25 2 4

25 2 4

730 7 8

Multiply by 9 Feet, first, as above directed; then, instead of multiplying by 8 Inches, let the 8 Inches be parted into such aliquot or even Parts of a Foot, as you find to be contain'd in that Figure, if you take such Parts of the Multiplicand, and add them to the former Product, the sum will give the Answer: Thus, 8 Inches may be parted into 4 and 4, because 4 is the third Part of 12. So, if you take the third Part of 75 Feet 7 Inches, and set it down twice, and add all together, the Sum will be 730 Feet 7 Inches 8 Parts, the same as before; thus, say how often 3 in 7, which is twice, set down 2; then, because twice 3 is 6, say, 6 out of 7 and there remains 1, for which you must add 10 to the 5, and it makes 15; then the Threes in 15 are 5 times, set down 5; and, because 3 times 5 is 15, there is 0 remains. Then go
to

Chap. IX. Multiplication of Feet, Inches, &c. 61

to the 7 Inches, saying, the Threes in 7 are twice; set down 2 in the Inches; and because twice 3 is but 6, take 6 out of 7, and there remains 1 Inch, which is 12 Parts; then the Threes in 12 are 4 times, and 0 remains. So the third Part of 75 Feet 7 Inches, is 25 Feet, 2 Inches, 4 Parts, which set down again, and add all together, the Sum is 750 Feet, 7 Inches, 8 Parts, the same as before.

Example 3. Let 97 Feet 8 Inches, be multiply'd by 8 Feet 9 Inches.

$$\begin{array}{r}
 97 \text{ } 8 \\
 \times 8 \text{ } 9 \\
 \hline
 781 \text{ } 4 \\
 48 \text{ } 10 \\
 24 \text{ } 5 \\
 \hline
 854 \text{ } 7
 \end{array}$$

Begin, first, to multiply by 8 Feet, saying, 8 times 8 is 64 Inches, that is, 5 Feet 4 Inches; set down 4 Inches, and carry 5, saying, 8 times 7 is 56, and 5 I carry is 61; set down 1, and carry 6, saying, 8 times 9 is 72, and 6 I carry is 78, which set down: Then, instead of multiplying by 9 Inches, take the aliquot Parts of 12 which 9 makes, which is 6 and 3, 6 Inches being half 12, and 3 the fourth Part; therefore take the half of 97 Feet 8 Inches, which is 48 Feet 10 Inches; and because 3 is half 6, you may take the half of 48 Feet 10 Inches, which is 24 Feet 5 Inches; add all up together, and the Sum is 854 Feet 7 Inches. See the Work as above.

G

Example

62 Multiplication of Feet, Inches, &c. Part I

Example 4. Let 75 Feet 9 Inches, be multiply'd by 17 Feet 7 Inches.

F.	I.	
75	9	
17	7	
525		
75		
35	13	P
18	11	3
8	6	
4	3	
1331	11	3

In this Example, because there are more than 12 Feet in the Multiplier, therefore I first multiply the 75 by 17 Feet; then, because the aliquot Parts in 7 Inches are 4 and 3, that is, a third and a fourth, I take the third Part of 75 Feet 9 Inches, which is 25 Feet 3 Inches, and the fourth Part thereof is 18 Feet, 11 Inches, 3 Parts; then the aliquot Parts of 9 Inches is 6 and 3, that is, half and a fourth; therefore I take half 17 Feet, which is 8 Feet 6 Inches, and the fourth Part is 4 Feet 3 Inches, (not meddling with the 7 Inches, because that was multiply'd into the 9 before;) then add all these together, and the Sum is 1331 Feet, 11 Inches, 3 Parts.

8	7	525
8	4	30
8	7	12
8	0	8

Example

Chap. IX. Multiplication of Feet, Inches, &c. 63

Example 5. Let 87 Feet 5 Inches, be multiply'd by 35 Feet 8 Inches.

F.	I.	
87	5	
35	8	
435		
261		
29	1	8
29	1	8
11	8	0
2	11	0
3117	10	4

Work here as in the last Example. After you have multiply'd the Feet, then take the aliquot Parts of 8 Inches, which is two Thirds; therefore take the third Part of 87 Feet 5 Inches, and set it down twice. Thus, the third Part of 87 Feet 5 Inches, is 29 Feet, 1 Inch, 8 Parts; set this down twice. Then the aliquot Parts of 5 Inches is 4 and 1, that is, a third Part and a twelfth Part; therefore take a third Part of 35, which is 11 Feet 8 Inches, and a twelfth Part of 35, is 2 Feet 11 Inches; set all these one under another, and add them together, and the Sum is 3117 Feet, 10 Inches, 4 Parts.

Example 6. Let 259 Feet 2 Inches, be multiply'd by 48 Feet 11 Inches.

F.	I.	
259	2	
48	11	
2072		
1036		
129	7	P
86	4	8
21	7	2
8	0	0
12677	6	10

G 2 First,

64 Multiplication of Feet, Inches, &c. Part I

First, Multiply the Feet; then take the aliquot Parts of 11, which will be 6, 4, and 1, that is, an half, a third, and a twelfth; therefore take the half of 259 Feet 2 Inches, which is 129 Feet 7 Inches, and a third Part is 86 Feet, 4 Inches, 8 Parts, and the twelfth Part of 259 Feet 2 Inches, is 21 Feet, 7 Inches, 2 Parts; or (because 1 is the fourth Part of 4) you may more readily take the fourth Part of 86 Feet, 4 Inches, 8 Parts, which is also 21 Feet, 7 Inches, 2 Parts; then add all together, and the Sum is 12677 Feet, 6 Inches, 10 Parts. See the foregoing Work.

I shall set down only the working of some few Examples in Feet and Inches, and then proceed to multiply Feet, Inches, and Parts, &c.

F.	I.	F.	I.
179	8	246	7
38	10	46	4
1432		1476	
537	P	984	P
89	7 6	82	2 4
59	9 0	15	4 0
9	6 0	11	6 0
Product	6960 10 6	Product	11425 0 4
F.	I.	F.	I.
246	7	257	9
36	9	39	11
1476		2313	
738	P	771	P
123	3 6	1128	10 6
61	7 9	85	11 0
12	0 0	215	5 9
9	0 0	19	6 0
Product	9061 11 3	Product	10288 6 3

Ex.

Chap. IX. Multiplication of Feet, Inches, &c. 65

Example 11. Let 7 Feet, 5 Inches, 9 Parts, be multiply'd by 3 Feet, 5 Inches, 3 Parts.

F.	I.	P.	
7	5	9	
3	5	3	
22	5	3	S
3	1	4	9 T
	1	10	5 3
25	8	6	2 3

In this Example, I first begin with 3 Feet, and thereby multiply 7 Feet, 5 Inches, 9 Parts: First, I say, 3 times 9 is 27 Parts, that is, 2 Inches and 3 Parts; set down 3 under the Parts, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Foot 5 Inches; set down 5 Inches, and carry 1, and say, 3 times 7 is 21, and 1 I carry is 22; set down 22 Feet. Then begin with 5 Inches, saying, 5 times 9 is 45, which is 45 Seconds, which make 3 Parts and 9 Seconds; set down 9 Seconds a Place towards the right Hand, and carry 3 Parts, saying, 5 times 5 is 25, and 3 I carry is 28, which is 2 Inches and 4 Parts; set down 4 Parts, and carry 2, saying, 5 times 7 is 35, and 2 I carry is 37, which is 3 Feet 1 Inch; set down 3 Feet 1 Inch, and begin to multiply by 3 Parts, saying, 3 times 9 is 27 Thirds, that is, 2 Seconds and 3 Thirds; set down 3 Thirds, and carry 2, saying, 3 times 5 is 15, and 2 I carry is 17, that is, 1 Part and 5 Seconds; set down 5 Seconds, and carry 1, saying, 3 times 7 is 21, and 1 I carry is 22, which is 1 Inch and 10 Parts, which set down, and add all up, and the Product is 25 Feet, 8 Inches, 6 Parts, 2 Seconds, 3 Thirds.

Note, That in multiplying Feet, Inches, and Parts, &c. if Feet be multiply'd by Feet, the Product is Feet; and Feet multiply'd by Inches, the Product is Inches;

66. *Multiplication of Feet, Inches, &c. Part I*

and the twelfth Part is Feet; and Parts multiply'd by Feet, the Product is Parts, and the twelfth Part thereof is Inches; Parts multiply'd by Inches, the Product is Seconds, and the twelfth Part thereof is Parts; and Parts multiply'd by Parts, the Product is Thirds, and the twelfth Part thereof is Seconds. So that if you begin to multiply Parts by Feet in the first Row, and Parts by Inches in the second Row, and Parts by Parts in the third Row, the first Figure in every Row will stand a Place more towards the right Hand, as you may see in the last Example.

Example 12. Let 37 Feet, 7 Inches, 5 Parts, be multiply'd by 4 Feet, 8 Inches, 6 Parts.

F.	I.	P.			
37	7	5			
4	8	6			
150	5	8	S		
12	6	5	8		
12	6	5	8	T	
1	6	9	8	6	
177	1	5	0	6	

First, I multiply by 4 Feet, saying, 4 times 5 is 20, which is 1 Inch 8 Parts; set down 8, and carry 1, saying, 4 times 7 is 28, and 1 I carry is 29; which is 2 Feet 5 Inches; set down 5 Inches, and carry 2, saying, 4 times 7 is 28, and 2 I carry is 30; set down 0, and carry 3, and say, 4 times 3 is 12, and 3 is 15; set down 15. Then I begin with 8 Inches; but because the Feet in the Multiplicand are more than 12, it will be the best Way to work for the aliquot Parts of 8; so here I work for 4 Inches, and set that down twice, 4 being the third Part of 12; therefore take the third Part of 37 Feet, 7 Inches, 5 Parts, which is 12 Feet, 6 Inches, 5 Parts, 8 Seconds; set this down twice. Then begin with 6 Parts; but instead of multiplying, take half

Chap. IX. Multiplication of Feet, Inches, &c. 67

37 Feet, 7 Inches, 5 Parts, (because 6 is half 12) and set it a Place more to the right Hand : Thus, the half of 37 Feet is 18, which I must count 18 Inches, because the Multiplier is 6 Parts; so the half of 37 Feet, 7 Inches, 5 Parts, is 1 Foot, 6 Inches, 9 Parts, 8 Seconds, 6 Thirds; which set down, and add all up together, and the Sum is 177 Feet, 1 Inch, 5 Parts, 0 Seconds, 6 Thirds.

Example 13. Let 311 Feet, 4 Inches, 7 Parts, be multiply'd by 36 Feet, 7 Inches, 5 Parts.

F.	I.	P.			
311	4	7			
36	7	5			
1866					
933			S		
103	9	6	4		
77	10	1	9	T	
8	7	9	6	4	
2	1	11	4	7	
12	0	0	0	0	
1	0	0	0	0	
	9	0	0	0	
11402	2	4	11	11	

In this Example, because the Feet in both the Multiplier and Multiplicand are compound Numbers, I first multiply the Feet one by the other; then take the aliquot Parts of 7 Inches, which are 4 Inches and 3, that is, a third and a fourth Part; so take the third Part of 311 Feet, 4 Inches, 7 Parts, which is 103 F. 9 I. 6 P. 4 S. and the fourth Part is 77 F. 10 I. 1 P. 9 Sec. set these down one under another, the Feet under the other Feet; then the aliquot Parts of 5 Parts are 4 and 1, that is, a third and a twelfth Part; so the third Part of 311 Feet, 4 Inches, 7 Parts, is 103 F. 9 I. 6 P. 4 Sec. but because the Multiplier is Parts, it must be set a Place to the right Hand, that

68 *Multiplication of Feet, Inches, &c. Part I*

that is, the 103 must be Inches, which is 8 Feet 7 Inches; therefore I set down 8 F. 7 I. 9 P. 6 Sec. 4 Th. Then, because 1 Inch is a fourth Part of 4 Inches; therefore I take a fourth Part of 8 F. 7 I. 9 P. 6 Sec. 4 Th. which is 2 F. 1 I. 11 P. 4 Sec. 7 Th. which is the same as if I had taken a twelfth Part of 311 F. 4 I. 7 P. Then, for 4 Inches in the Multiplicand, instead of multiplying 36 Feet by it, take a third Part, because 4 Inches is a third Part of 12; so the third Part of 36 is 12 Feet, and the aliquot Parts of 7 Parts, is 4 and 3, that is, a third and a fourth; so the third Part of 36 is 12, which now is 12 Inches, that is, 1 Foot, and the fourth Part is 9 Inches; add all these together, and the Sum will be 11402 Feet, 2 Inches, 4 Parts, 11 Seconds, 11 Thirds.

Example 14. Let 8 Feet, 4 Inches, 3 Parts, 5 Seconds, 6 Thirds, be multiply'd by 3 Feet, 3 Inches, 7 Parts, 8 Seconds, 2 Thirds.

	F.	I.	P.	S.	T.
	8	4	3	5	6
	3	3	7	8	2
25	0	10	4	6	
2	1	0	10	4	6
	4	10	6	0	2
		5	6	10	3
			1	4	8
Product	27	7	3	5	1

In this last Example there is no Difficulty, if you do but observe the former Directions, and set every Row a Place more to the right Hand.

I shall

Chap. IX. Multiplication of Feet, Inches, Sec. 69

I shall only set down the Working of some few Examples more, and so conclude this Chapter.

F.	I.	P.	F.	I.	P.
321	7	3	42	7	8
9	3	6	7	3	6

F.	I.	P.	F.	I.	P.
2894	5	3	298	5	8
80	4	9	10	7	11
13	4	9	1	9	3

F.	I.	P.	F.	I.	P.
2988	2	10	310	10	10
	4	6		10	10

F.	I.	P.	F.	I.	P.
114	7	9	139	10	8
14	6	1	18	7	4

F.	I.	P.	F.	I.	P.
496			2072		
124		S	259		S
62	3	10	86	7	6
1	8	9	21	7	10
7	0	0	7	2	7
1	2	0	9	0	0
7	0	0	6	0	0
3	6	0	1	0	0
1809	1	1	4793	6	0
	9	6		10	8

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F.	I.	P.	F.	I.	P.
267	7	10	317	9	7
15	9	7	37	5	9

1335	2219
534	951
133 9 11 S	105 11 2 4
66 10 11 6	26 5 9 7 T
11 1 9 11 T	13 2 10 9 6
11 10 3 7 1	6 7 5 4 9
12 6 0 0 0	18 6 0 0 0
2 1 0 0 0	9 3 0 0 0
1 0 6 0 0	1 6 6 0 0
8 1 0 0	3 1 0 0
6905 0 7 0 10	1915 9 11 1 3

CHAP. I



A SQUARE is a Geometrical Figure, having four equal sides, many right (or square) Angles. To find the superficial Content thereof, this

THE RULE

Multiply the Side into itself, and the Product is

THE



THE
Compleat Measurer, &c.

PART II.

CHAP. I.

The Mensuration of SUPERFICIES.

Superficial Figures are all such as have only Length and Breadth, not having any commensurable Thickness.

§ I. Of a SQUARE.

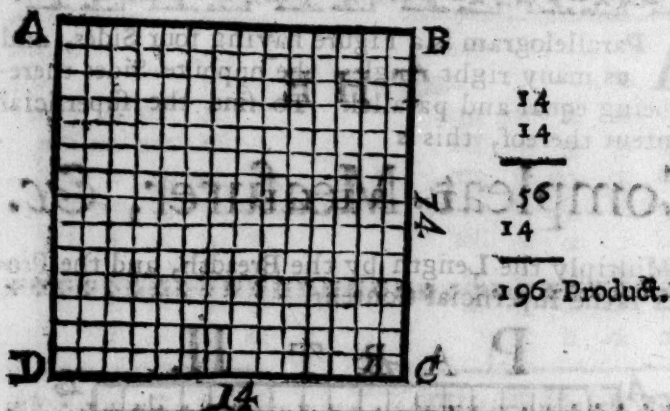
A SQUARE is a Geometrical Figure, having four equal Sides, and as many right (or square) Angles. To find the superficial Content thereof, this is

The RULE.

Multiply the Side into it self, and the Product is the Content.

Let

Let ABCD be a Geometrical Square given, each Side being 14 Feet, Yards, Poles, or other Measure; multiply 14 by it self, and the Product is 196, which is the superficial Content.



By Scale and Compasses.

Extend the Compasses from 1 in the Line of Numbers, to 14; the same Extent will reach from the same Point, turn'd forward to 196.

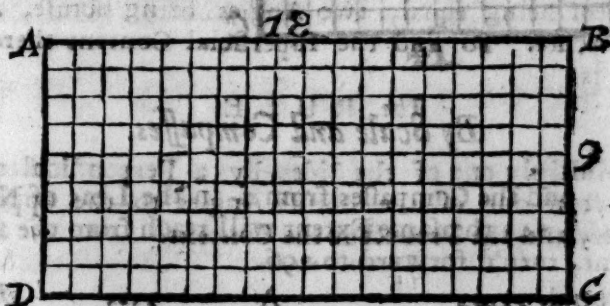
Demonstration. Let each Side of the given Square be divided into 14 equal Parts, and Lines drawn from one another, crossing each other within the Square; so shall the whole great Square be divided into 196 little Squares, as you may see in the Figure above, equal to the Number of square Feet, Yards, or Poles, or other Measures, by which the Side was measur'd.

**§ II. Of a PARALLELOGRAM,
or Long Square.**

A Parallelogram is a Figure having four Sides, and as many right Angles, the opposite Sides thereof being equal and parallel. To find the superficial Content thereof, this is

The RULE.

Multiply the Length by the Breadth, and the Product is the superficial Content.



Length 18

Breadth 9

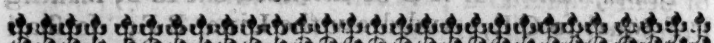
Prod. 162

Let ABCD be a long Square, the Length thereof 18 Feet, and the Breadth 9 Feet, which multiply'd together, the Product is 162, the superficial Content thereof.

By Scale and Compasses.

Extend the Compasses in the Line of Numbers from 1 to 9, the same Extent will reach from 18 down to 162, the Square Feet.

Demonstration. If the Sides, A B and C D, be each divided into 18 equal Parts, representing 18 Feet; and the Lines A D and B C each divided into 9 equal Parts, and Lines drawn from Point to Point, crossing each other within the Figure; those Lines will make thereby so many little Squares as there are Square Feet, viz. 162.



§ III. Of a RHOMBUS.

A Rhombus is a Figure representing a Quarry of Glass, having four equal Sides, the opposites thereof being equal, two Angles being obtuse, and two acute. To find the superficial Content thereof, this is

The R U L E.

Multiply one of the Sides by a Perpendicular let fall from one of the obtuse Angles to the opposite Sides, and the Product is the Content.

Perpend. 13.42

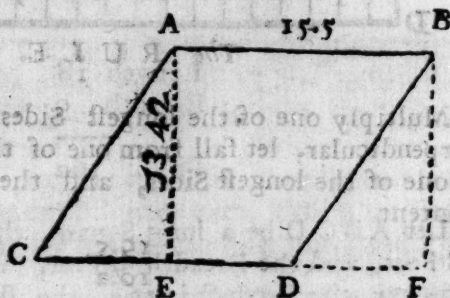
The Side 15.5

6710

6710

1342

Prod. 208.010



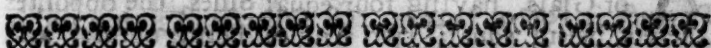
Let A B C D be a Rhombus given, whose Sides are each 15.5 Feet, and the Perpendicular E C is 13.42, which multiply'd together, the Product is 208.01; which is the superficial Content of the Rhombus, that is, 208 Feet and one hundredth Part of a Foot.

By

By Scale and Compasses.

Extend the Compasses from 1 to 13.42, that Extent will reach from 15.5, the same Way to 208 Feet, the Content.

Demonstration. If A B be extended out to E, making D F equal to C E, and draw the Line B F; so shall the Triangle D B F be equal to the Triangle A C E : For D F and C E are equal, and B F is equal to A E, because A B and C F are parallel. Therefore the Parallelogram A B E F is equal to the Rhombus A B C D.

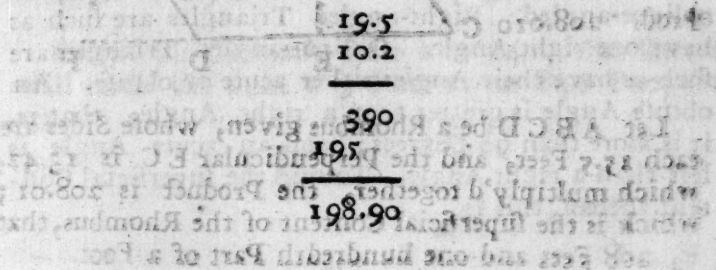


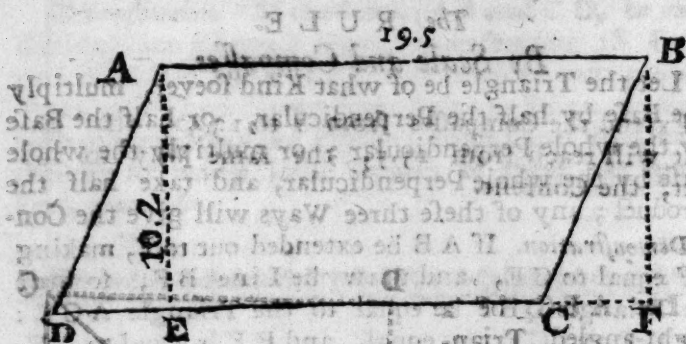
§ IV. Of a RHOMBOIDES.

A Rhomboides is a Figure having four Sides, the opposite whereof are equal and parallel; and also four Angles, the opposite thereof are equal. To find the superficial Content thereof, this is

The R U L E.

Multiply one of the longest Sides thereof by the Perpendicular, let fall from one of the obtuse Angles to one of the longest Sides, and the Product is the Content.





Let A B C D be a Rhomboides given, whose longest Sides, A B or C D, is 19.5 Feet, and the Perpendicular A E is 10.2; which multiply'd together, the Product is 198.9, that is, 198 superficial Feet, and 9 tenth Parts, the Content.

Demonstration. If D C be extended to F, making C F equal to D E, and a Line drawn from B to F; so will the Triangle C B F be equal to the Triangle A D E, and the Parallelogram A E F B be equal to the Rhomboides A B C D, which was to be prov'd.



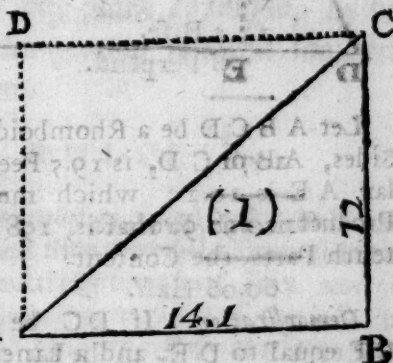
§ V. Of a TRIANGLE.

A Triangle is a Figure having three Sides and three Angles. Triangles are either right-angled or oblique-angled. Right-angled Triangles are such as have one right Angle. Oblique-angled Triangles are such as have their Angles either acute or obtuse. An obtuse Angle is greater than a right Angle, that is, it is more than 90 Degrees; and an acute Angle is less than a right Angle. To find the superficial Content thereof, this is.

The R U L E.

Let the Triangle be of what Kind soever, multiply the Base by half the Perpendicular, or half the Base by the whole Perpendicular; or multiply the whole Base by the whole Perpendicular, and take half the Product; any of these three Ways will give the Content.

Let ABC be a right-angled Triangle, whose Base is 14.1 Feet, and the Perpendicular 12 F. Multiply 14.1 by 6, half the Perpendicular, and the Product is 84.6 Feet, the Content. Or multiply A 14.1 by 12, the Product is 169.2; the half thereof is 84.6, the same as before.



14.1 Base.
6 half Perpendicular.

84.6 Product.

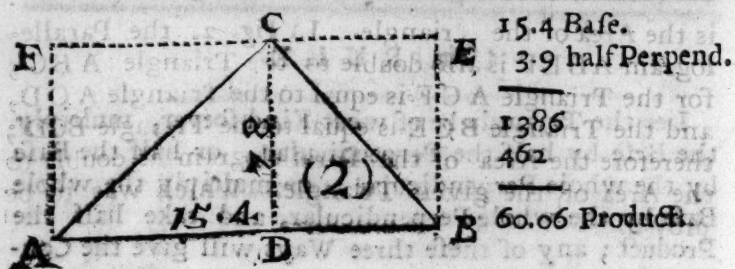
14.1 Base.
12 Perpendicular.

169.2 Product.

84.6 Half.

By Scale and Compasses.

Extend the Compasses from 2 to 14.1, that Extent will reach the same Way from 12 to 84.6 Feet, the tent.



15.4 Base.

7.8 Perpend.

1232

1078

120.12 Product.

60.06 Half.

15.4 Base.

3.9 half Perpend.

1386

462

60.06 Product.

7.7 half Base.

7.8 Perpend.

816

539

60.06 Product.

Let ABC (Fig. 2) be an oblique-angled Triangle given, whose Base is 15.4, and the Perpendicular 7.8; if 15.4 be multiply'd by 3.9, (half the Perpendicular) the Product will be 60.06 for the Area or superficial Content: Or if the Perpendicular 7.8 be multiply'd into half the Base 7.7, the Product will be 60.06, as before: Or if 15.4, the Base, be multiply'd by the whole Perpendicular 7.8, the Product will be 120.12, which is the double Area; the half thereof is 60.06 Feet, as before. See the Work.

By Scale and Compasses.

Extend the Compasses from 2 to 15.4, that Extent will reach from 7.8 to 60 Feet, the Content.

Demonstration. If AD (Fig. 1.) be drawn parallel to BC , and DC parallel to AB ; so shall the Triangle ADC be equal to the given Triangle ABC . Hence the Parallelogram $ABCD$ is double to the given Triangle; therefore half the Area of the Parallelogram

is

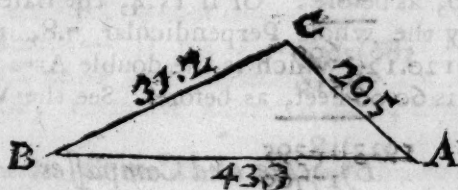
is the Area of the Triangle. In Fig. 2. the Parallelogram ABEF is also double to the Triangle ABC; for the Triangle ACF is equal to the Triangle ACD, and the Triangle BCE is equal to the Triangle BCD; therefore the Area of the Parallelogram is double to the Area of the given Triangle. Which was to be prov'd.

To find the Area of any plain Triangle, by having the three Sides given, without the Help of a Perpendicular.

The RULE.

Add the three Sides together, and take half that Sum; then subtract each Side severally from that half Sum: Which done, multiply that half Sum and the three Differences continually, and out of the last Product extract the square Root, which square Root shall be the Area of the Triangle sought.

Example. Let ABC be a Triangle, whose three Sides are as followeth, viz. AB 43.3, AC 20.5, and BC 31.2, the Area is requir'd.



Sides	$\begin{array}{r} 43.3 \\ 31.2 \\ 20.5 \end{array}$	$\begin{array}{r} 42 \\ 16.3 \\ 27.0 \end{array}$	Differences.
	Sum	95.0	
	Half	47.5	

Area

Area 296.31

47.5 The half Sum.

27 Difference.

3325

950

1282.5 Product.

16.3 Difference.

38475

76950

12825

20904.75 Product.

4.2 Difference.

4180950

8361900

87799.9500(296.31

4

49)477

441

586)3699

3516

5923)18395

17769

59261) 61600

59261

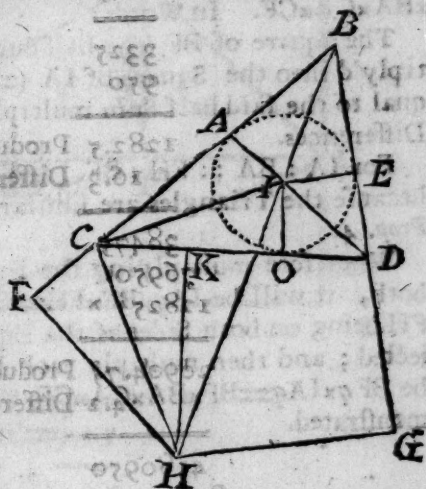
3339 Remains.

Demonstration. In the Triangle BCD, I say, if from the half Sum of the Sides, you subtract each particular Side, and multiply the half Sum and the three Differences together continually, the square Root of the Product shall be the Area of the Triangle.

First, by the Lines BI, CI, and DI, bisect the three Angles, which Lines will all meet in the Point I; by which Lines the given Triangle is divided into three new Triangles, CBI, DCI, and BDI; the Perpendiculars of which new Triangles, are the Lines AI, EI, and OI, being all equal to one another, because the Point I is the Center of the inscrib'd Circle, (by *Euclid, Lib. 4. Prop. 4.*) Wherefore to the Side BC join CF equal to DE, or DO; so shall BF be equal to half the Sum of the Sides, viz. $= \frac{1}{2} BC + \frac{1}{2} BD + \frac{1}{2} CD$.

And $BA = BF - CD$ for $CA = CO$ and $OD = CF$, therefore $CD = AF$ and $AC = BF - BD$ for $BE = BA$ and $ED = CF$, therefore $BD = BA + CF$, and $CF = BF - BC$.

Then make $CK = CF$, and draw the Perpendiculars FH, GH, and KH, and extend BI to H; because the Angles FCK + FHK are equal to two right Angles, (for the Angles F and K are right Angles) equal also to FCK + ACO, (by *Euclid I. 13.*) And the Angles ACO + AIO are equal to two right Angles; therefore the Quadrangles FGKH and AIOC are alike; and the Triangles CFH and AIC are also similar; and the Triangles BAI and BFH are likewise similar.



From this Explanation, I say, the Square of the Area of the given Triangle will be $BF \times IA = BF \times BA \times CA \times CF$. In Words:

The Square of BF (the half Sum of the Sides) multiply'd into the Square of IA ($= IF = IO$) will be equal to the said half Sum multiply'd into all the three Differences.

For $IA : BA :: FH : BF$, and $IA : CF :: AC : FH$; because the Triangles are similar. By *Euclid, Lib. 6, Prop. 4.*

Wherefore multiplying the Extremes and Means in both, it will be $IA \times BF \times FH = BA \times CA \times CF \times FH$; but FH being on both Sides of the Equation, it may be rejected; and then multiply each Part by BF, it will be $BF \times IA = BF \times BA \times CA \times CF$. Which was to be demonstrated.



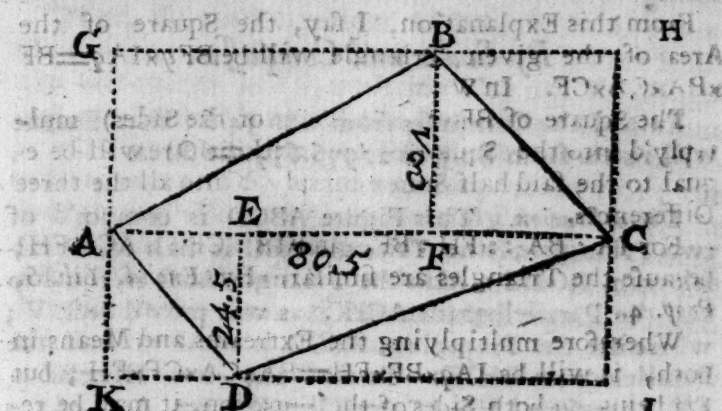
§ VI. Of a TRAPEZIUM.

A Trapezium is a Figure having four unequal Sides and oblique Angles. To find the Area or superficial Content thereof, this is

The R U L E.

Add the two Perpendiculars together, and take half the Sum, and multiply that half Sum by the Diagonal, or multiply the whole Sum by half the Diagonal, the Product is the Area. Or you may find the Area's of the two Triangles, ABC and ACD, (by Section V) and add those two Area's together, the Sum shall be the Area of the Trapezium.

$$BF = 30.1$$



$$BF = 30.1$$

$$DE = 24.5$$

$$\text{Sum } 54.6$$

$$\text{Half } 27.3$$

$$AC = 80.5$$

$$1365$$

$$2184$$

$$\text{Area } 2197.65$$

Let ABCD be a Trapezium given, the Diagonal whereof is 80.5, and the Perpendicular BF 30.1, and the Perpendicular DE 24.5, these two added together, the Sum is 54.6, the half thereof is 27.3, which multiply'd by the Diagonal 80.5, the Product is 2197.65, which is the Area of the Trapezium; or if 40.25, half the Diagonal, be multiply'd by 54.6, the whole Sum of the Perpendiculars, the Product is 2197.65, the same as before.

By Scale and Compasses.

Extend the Compasses from 2 to 54.6; that Extent will reach from 80.5 to 21976.65, the Area.

Demonstration. This Figure ABCD is compos'd of two Triangles; the Triangle ABC is half the Parallelogram AGHC: Also the Triangle ACD is equal to half the Parallelogram ACIK, as was prov'd Sect. V; wherefore the Trapezium ABCD is equal to half the Parallelogram GHIK. To find the Area $HI=BF+DE$, therefore $\frac{1}{2} HI \times AC (=KI=GH) =$, Area of the Trapezium. Which was to be prov'd.

§ VII. Of Irregular FIGURES.

Irregular Figures are all such as have more Sides than four, and the Sides and Angles unequal. All such Figures may be divided into as many Triangles as there are Sides wanting two. To find the Area of such Figures, they must be divided into Trapeziums and Triangles, by Lines drawn from one Angle to another, and so find the Area's of the Trapeziums and Triangles severally, and then add all the Area's together, so will you have the Area of the whole Figure.

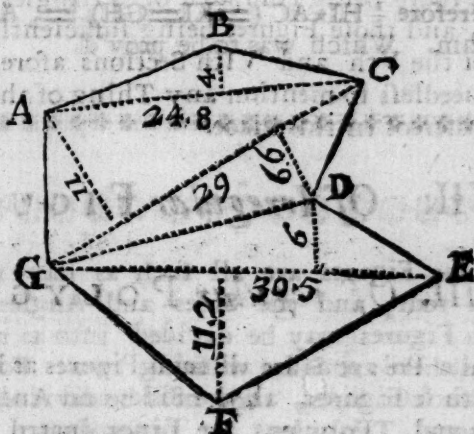
Let ABCDEFG be an irregular Figure given to be measur'd; first, draw the Lines AC and GD, and thereby divide the given Figure into two Trapeziums, ACGD and GDEF, and the Triangle ABC; of all which I find the Area's severally.

First, I multiply the Base AC by half the Perpendicular, and the Product is 49.6, the Area of the Triangle ABC.

Then

Then, for the Trapezium ACGD, the two Perpendiculars, 11 and 6.6, added together, make 17.6; the half thereof is 8.8, multiply'd by 29, the Diagonal, the Product is 255.2, the Area of that Trapezium.

And for the Trapezium GDEF, the two Perpendiculars, 11.2 and 6, added together, make 17.2; the half thereof is 8.6, which multiply'd by 30.5, the Diagonal, the Product is 262.3, the Area thereof. All these Area's added together, make 567.1, and so much is the Area of the whole irregular Figure. See the Work.



24.8 Base AC.
2 half Perpendicular.

49.6 Area of ABC.

11
6.6 } Perpendiculars;

17.6 Sum.

8.8 half.

29 Diag. CG.

792

176

255.2 Area of ACGD.

11.2	} Perpendiculars.	30.5	
6.		8.6	
17.2	Sum.	1830	
8.6	half Sum.	2440	
		262.30	Area of GDEF.
		255.2	Area of ACGD.
		49.6	Area of ABC.
		567.1	Sum of the Area's.

This Figure being compos'd of Triangles and Trapeziums, and those Figures being sufficiently demonstrated in the Vth and VIth Sections aforegoing, it will be needless to mention any Thing of the Demonstration thereof in this Place.



§ VIII. Of regular POLYGONS.

Regular Polygons are all such Figures as have more than four Sides, all the Sides and Angles thereof being equal. Polygons are denominated from the Number of their Sides and Angles.

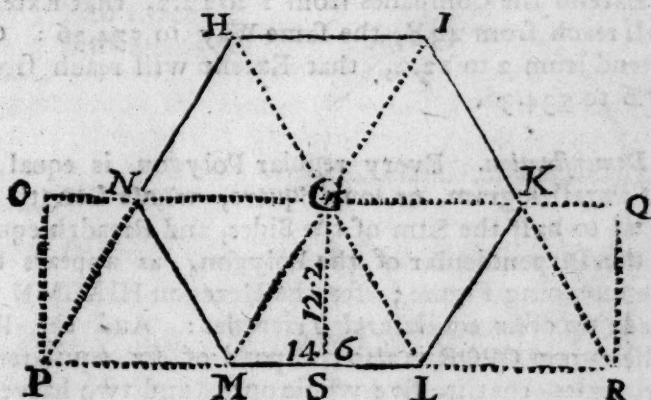
If the Figure consists of	{	5	Equal Sides and An- gles, it is call'd a regular	{	Pentagon.
		6			Hexagon.
		7			Heptagon.
		8			Octagon.
		9			Enneagon.
		10			Decagon.
		11			Endecagon
		12			Dodecagon

Chap. I. *Mensuration of Superficies.* 87

To find the Area or superficial Content of any regular Polygon, this is

The RULE.

Multiply the whole Perimeter, or Sum of the Sides, by half the Perpendicular, let fall from the Center to the Middle of one of the Sides; or multiply the half Perimeter by the whole Perpendicular, and the Product is the Area.



14.6

3

43.8 half Sum of the Sides.

12.2 the Perpendicular.

876
876
438
534.36 Area.

14.6

6

87.6 The Sum of the Sides.

6.1 Half the Perpendicular.

876
5256
534.36 Area.
I 2

Let

Let HIKLMN be a regular Hexagon, each Side thereof being 14.6, the Sum of all the Sides is 87.6, the half Sum thereof is 43.8, which multiply'd by the Perpendicular GS 12.2, the Product is 534.36: Or if 87.6, the whole Sum of the Sides be multiply'd by half the Perpendicular 6.1, the Product is 534.36, the same as before, which is the Area of the given Hexagon.

By Scale and Compasses.

Extend the Compasses from 1 to 12.2, that Extent will reach from 43.8, the same Way to 534.36: Or extend from 2 to 12.2, that Extent will reach from 87.6 to 534.36.

Demonstration. Every regular Polygon is equal to the Parallelogram, or long Square, whose Length is equal to half the Sum of the Sides, and Breadth equal to the Perpendicular of the Polygon, as appears by the foregoing Figure; for the Hexagon HIKLMN is made up of six equilateral Triangles: And the Parallelogram OPQR is also compos'd of six equilateral Triangles, that is, five whole ones, and two halves; therefore the Parallelogram is equal to the Hexagon.

§ IX. Of a CIRCLE.

A Circle is a plain Figure, contain'd under one Line, which is call'd a Circumference, unto which all Lines drawn from a Point in the Middle of the Figure call'd the Center, and falling upon the Circumference thereof, are all equal the one to the other. The Circle contains more Space than any plain Figure of equal Compass.

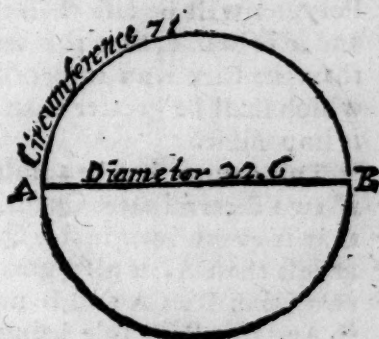
Problem

Problem 1. Having the Diameter and Circumference, to find the Area.

The R U L E.

Every Circle is equal to a Parallelogram, whose Length is equal to half the Circumference, and the Breadth equal to half the Diameter; therefore multiply half the Circumference by half the Diameter, and the Product is the Area of the Circle.

35.5	Half Circumf.
11.3	Half Diameter.
1065	
355	
355	
401.15	Area.



Thus, if the Diameter of a Circle (that is, the Line drawn cross the Circle through the Center) be 22.6, and if the Circumference be 71, the half of 71 is 35.5, and the half of 22.6 is 11.3; which multiply'd together, the Product is 401.15, which is the Area of the Circle.

Demonstration. Every Circle may be conceiv'd to be a Polygon of an infinite Number of Sides, and the Semidiameter must be equal to the Perpendicular of such a Polygon, and the Circumference of the Circle equal to the Periphery of the Polygon; therefore half the Circumference, multiply'd by half the Diameter, gives the Area as aforesaid.

Or, (with *F. Ignat. Gascon Pardies*) " Every Circle is
 " equal to a Rectangle-Triangle, one of whose Legs
 " is the Radius, and the other a right Line equal to
 I 3 " the

“ the Circumference of the Circle : For such a Triangle will be greater than any Polygon inscrib'd, and less than any Polygon circumscrib'd, (by the 24th, 25th, 26th, and 27th Articles of the fourth Book of his Elements of Geometry) and therefore must be equal to the Circle.

“ For (says he) should it be greater than the Circle, be the Excess as little as it will, a Polygon may be circumscrib'd, whose Difference from the Circle shall be yet less than the Difference between that Circle and the Rectangle-Triangle; and that that Polygon will be less than the Triangle, is absurd; and if it be said, that this rectangled Triangle is less than the Circle, an inscrib'd Polygon may be made, which shall be greater than that Triangle; which is impossible.

“ This cannot but be admitted as a Principle, That if two determinate Quantities A and B are such, that if every imaginable Quantity which is greater or less than A, is also greater or less than B, these two Quantities A and B must be equal.

“ And this Principle being granted, which is in a Manner self-evident, it may directly be prov'd, that the Triangle (before mention'd) is equal to the Circle; because every imaginable inscrib'd Figure which is less than the Circle, is also less than the Triangle; and every circumscrib'd Figure greater than the Circle, is also greater than the Triangle.

Problem 2. Having the Diameter of a Circle, to find the Circumference.

As 7 to 22, so is the Diameter to the Circumference.
Or, as 113 to 355, so is the Diameter to the Circumference.

Or, as 1 to 3.141593, so is the Diameter to the Circumference.

Let the Diameter (as in the former Circle) be 22.6, this multiply'd by 22, and the Product is 497.2; which divided by 7, gives 71.028 for the Circumference.

rence. Or, (by the second Proportion) if 22.6 be multiply'd by 355, the Product will be 8023.; this divided by 113, the Quotient is 71, the Circumference. Or, (by the third Proportion) if 22.6 be multiply'd into 3.141593, the Product is 71.0000018, the Circumference; which two last Proportions are the most exact.

$ \begin{array}{r} 22.6 \\ 22 \\ \hline 452 \\ 452 \\ \hline 7)497.2(71.028 \end{array} $	$ \begin{array}{r} 355 \\ 22.6 \\ \hline 2130 \\ 710 \\ 710 \\ \hline 113)8023.0(71 \\ 791 \\ \hline 113 \\ 113 \\ \hline \dots \end{array} $
$ \begin{array}{r} 3.141593 \\ 22.6 \\ \hline 18849558 \\ 6283186 \\ 6283186 \\ \hline 71.0000018 \end{array} $	

By Scale and Compasses.

Extend the Compasses from 7 to 22, or from 113 to 355, or from 1 to 3.14159; that Extent will reach from 22.6 to 71.

The Proportion of the Diameter of a Circle, to the Circumference, was never yet exactly found, notwithstanding many eminent learned Men have labour'd very far therein; amongst which, the excellent *Van Culen* hath hitherto out-done all, in his having calculated the said Proportion to 36 Places of Decimals, which are engraven upon his Tomb-stone in *St. Peter's Church in Leyden*; which Numbers are these:

Diameter.

Diameter.

1.00000.00000.00000.00000.00000.00000.00000

Circumference.

3.14159.26535.89793.23846.26433.83279.50288

Of which large Number theſe fix Places, 3.14159, anſwering to the Diameter 1.00000, may be ſufficient; of the three Proportions, as 7 to 22, 113 to 355, and 1 to 3.14159, I ſhall leave my Reader to uſe which of them he pleaſes, but ſhall commend the laſt two as moſt exact, tho' the firſt be moſt in Uſe; but in the following Work I ſhall uſe ſometimes one of them, and ſometimes another, but for the moſt Part that of *Van Cnlen*, as being moſt exact.

Problem 3. Having the Circumference of a Circle, to find the Diameter.

As 1 is to .318309, ſo is the Circumference to the Diameter.

Or, As 355 to 113, ſo is the Circumference to the Diameter.

Or, As 22 to 7, ſo is the Circumference to the Diameter.

Let the Circumference be 71, (as in the former Circle) if .318309 be multiply'd by 71, (as by the firſt Proportion) the Product will be 22.59939 for the Diameter. Or, by the ſecond Proportion, 113 multiply'd by 71, the Product is 8023; which divided by 355, the Quotient will be 22.6, the Diameter. Or, by the third Proportion, 71 multiply'd by 7, the Product is 497; this divided by 22, the Quotient is 22.5909, the Diameter.

.318309

.318309	113	71
71	71	7
318309	113	22)497(22.59
2228163	791	44
22.599939	355)8023(22.6	57
	710	44
	923	130
	710	110
	2130	200
	2130	198
		2

Thus, by both the first Portions, the Diameter is 22.6, but by the last it falls something short.

By Scale and Compasses.

Extend the Compasses from 3.14159 to 1, that Extent will reach from 71 to 22.6, which is the Diameter sought.

Or you may extend from 1 to .318309.

Or from 22 to 7.

Or from 355 to 113; the same will reach from 71 to 22.6, as before.

Note. That if the Circumference be 1, the Diameter will be .318309.

Problem 4. Having the Diameter of a Circle, to find the Area.

All Circles are in Proportion one to another, as are the Squares of their Diameters, (by *Euc.* 12. 2.) Now, the Area of a Circle whose Diameter is 1, will be .785398, according to *Van Culen's* Proportion before

fore mention'd; but for Practice .7854 will be sufficient: Therefore,

As 1 (the Square of the Diameter 1) is to .7854, so is 510.76 (the Square of 22.6, the Diameter of the given Circle) to 401.15, (the Area of the given Circle :) But,

According to *Metius's* Proportion,

As 452 : 355 :: 510.76 : 401.15, the same as before.

But if you use *Archimedes's* Proportion, say,

As 14 : 11 :: 510.76 : 401.31; which Area is greater than by the two former Proportions; though in small Circles this is near enough the Truth. See the Working of all these.

22.6 Diameter of the former Circle.

22.6

1356

452

452

510.76 The Square of the said Diameter.

As 1 : .7854 :: 510.76

.7854

204304

255380

408608

357532

401.150904 The Area.

By Scale and Compasses.

The Extent from 1 to 22.6, being twice turn'd over from .7854, will fall at the last upon 401.15, the Area.

113

4

452

As 452 : 355 :: 510.76

355

255380

255380

153228

452)181319.80(401.15

1808

519

452

678

452

2260

2260

As

As 14 : 11 :: 510.76

11

14)561836(401.31

56

18

14

43

42

16

14

2

Problem 5. Having the Circumference of a Circle, to find the Area.

Because the Diameters of Circles are proportional to their Circumferences; that is, as the Diameter of one Circle is to its Circumference; so is the Diameter of another Circle to its Circumference. Therefore the Area's of Circles are to one another, as the Squares of the Circumferences. And if the Circumference of a Circle be 1, the Area of that Circle will be .07958; then the Square of 1 is 1, and the Square of 71 (the Circumference of the former Circle) is 5041. Therefore it will be,

Sq. Cir. Area Sq. Circumf.

As 1 : .07958 :: 5041

5041

7958

31832

397900

401.16278 Area.

12

4

88

Or

Or thus :

$$\text{As } 88 : 7 :: 5041$$

7

$$88)35287(400.98 \text{ Area.}$$

352

870

792

780

704

76

Area.

$$\text{Or, As } 1420 : 113 :: 5041 : 401.15$$

Problem 6. By having the Diameter, to find the Side of a Square that is equal in Area to that Circle.

If the Diameter of a Circle be x , the Side of a Square equal theretunto will be .8862. Therefore,

$$\text{As } 1 : .8862 :: 22.6 \text{ (the Diameter.)}$$

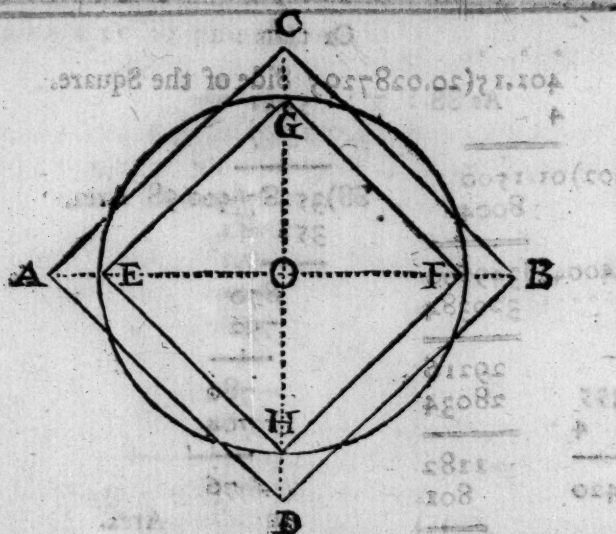
22.6

53172

17724

17724

To 20.02812 the Side of the Square AC.



Let the Diameter of the Circle EF, or GH, be 22.6, (as before) to find the Side of the Square AC, AD, &c. If .8962 be multiply'd by 22.6, the Product is 20.02812, which is the Side of a Square, equal in Area to the Circle given; for if 20.02812 be multiply'd square-wise, that is, by it self, it will produce 401.1255907344, which is nearly equal to the Area found in the last Problem.

You may find the Side of the Square equal, by extracting the square Root out of the Area of the given Circle.

401.15 (20.0287295 Side of the Square.

4

4002) 01.1500

8004

40048) 349600

... 320384

29216

28034

1182

8014

381

360

11

20

11

N. B. By this Method of extracting the Square Root of the Area, you may find the Side of a Square equal to any plain Figure, regular or irregular.

Problem 7. By having the Circumference, to find the Side of the Square equal.

If the Circumference of a Circle be 1, the Side of the Square equal will be .2821. Therefore,

As 1 : .2821 :: 71 (the Circumference.)

71

2821

19747

To 20.0291 the Side of the Square.

K 2

Problem

Problem 8. Having the Diameter, to find the Side of a Square which may be inscrib'd in that Circle.

If the Diameter of a Circle be 1, the Side of the Square inscrib'd will be .7071. Therefore,

$$\text{As } 1 :: .7071 :: 22.6$$

22.6

42426

14142

14142

To 15.98046 the Side EG inscrib'd.

Or, if you square the Semidiameter, and double that Square, the square Root of the doubled Square will be the Side of the Square inscrib'd. For (by *Euclid* 1. 47) the Square of the Hypotenuse EG is equal to the Sum of the other two Legs, EO and OG.

11.3 Semidiameter.

11.3

339

113

113

127.69 the Square of EO, which double, because EO=OG.

255.38 (15.98 Root, which is the Side of the Squ.

1

25) 155

125

309) 3038

2781

3188) 25700

25504

196

Problem

Problem 9. Having the Circumference, to find the Side of a Square which may be inscrib'd.

If the Circumference be 1, the Side of the Square inscrib'd will be .2251. Therefore,

$$\text{As } 1 : .2251 :: 71$$

71

2251

1757

To 15.9821 the Side of the Square EG.

Because that in each of the four last Problems, viz. the 6th, 7th, 8th, and 9th, there is a Proportion laid down, it will be easy to work them with Scale and Compasses; for if you extend the Compasses from the first to the second, that Extent will reach from the third to the fourth. As in the last Problem, where the Proportion is as 1 to .2251, so is 71 to the Side of the Square 15.9821. Here extend the Compasses from 1 to .2251; that Extent will reach from 71 to 15.98; and so of the rest. But the 5th must be wrought like the 4th, thus; extend the Compasses from 1 to 71; that Extent, turn'd twice over the same Way from .07958, will fall, at the last, upon 401.15.

Problem 10. Having the Area, to find the Diameter.

If the Area of a Circle be 1, the Square of the Diameter thereof is 1.2732. Therefore,

K 3

As

Area. Sq. Diam. Area.

As 1 : 1.2732 :: 401.15

401.15

63660

12732

12732

50928

510.744189 the Diameter.

4

42)110

84

445)2674

2225

4509)44941

40581

45189)436080

406701

29379

By Scale and Compasses.

Extend the Compasses from 1 to 1.2732; that Extent will reach from 401.15 to 510.74, &c. Then divide the Space between 1 and 510.74 into two equal Parts, and you will find the middle Point at 22.6. Or you may divide the Space upon the Line of Numbers, between 401.55 and .7854, into two equal Parts, and one of those Parts will reach from 1 to 22.6, the Diameter sought.

Problem

Problem 11. Having the Area, to find the Circumference.

If the Area of a Circle be 1, the Square of the Circumference will be 12.56637. Therefore,

$$\begin{array}{r} \text{Ar. Sq. Circumf. Area.} \\ \text{As } 1 : 12.56637 :: 401.15 \\ \hline 401.15 \end{array}$$

$$\begin{array}{r} 6283185 \\ 1256637 \\ \hline 1256637 \\ 5026548 \end{array}$$

$$\begin{array}{r} \dots \dots \text{Circumf.} \\ 5040.99932550(70.9999 \text{ Root,} \\ 49 \end{array}$$

$$\begin{array}{r} 1409)14099 \\ 12681 \\ \hline \end{array}$$

$$\begin{array}{r} 14189)141893 \\ 127701 \\ \hline \end{array}$$

$$\begin{array}{r} 141989)1419225 \\ 1277901 \\ \hline \end{array}$$

$$\begin{array}{r} 1419989)14132450 \\ 12779901 \\ \hline 1352549 \end{array}$$

By Scale and Compasses.

Divide the Space between 401.15 and .07958, upon the Line, into two equal Parts; one of those Parts will reach from 1 to 71, the Circumference sought.

Problem 12. Having the Area, to find the Side of a Square inscrib'd.

If the Area of a Circle be 1, the Area of a Square inscrib'd within that Circle will be .6366. Therefore,

As 1 : 401.15 :: .6366

.6366

240690
240690

120345

240690

255.372090 (15.98 Root, which is the Side of the Square sought.)

25)155

125

309)3037

2781

3188)25620

25504

11690

The same Reason may be given for the last Proportion, that was given before for the Proportion of Circles to the Squares of their Diameters and Circumferences; for not only the Squares of the Diameters and Circumferences are in Proportion to the Circles they belong to; but also all Figures inscrib'd or circumscrib'd, have the Squares of their like Sides proportional to the Circles they are inscrib'd in, or circumscrib'd about; and also to the Figures themselves: The Square of any Side of one Figure is to the Area of that

that Figure, as the Square of the like Side of another similar Figure is to the Area thereof, as you may find prov'd at large in *Euclid, Sturmius, Mathesis Enumerata*, and other Authors, but will be too large to insert in this Place.

By Scale and Compasses.

Extend the Compasses from 1 to 401.15; that Extent will reach from .6366 to 255.37; the half Space between that and 1 is at 15.98, the Side of the Square.

Problem 13. Having the Side of a Square, to find the Diameter of the circumscribing Circle.

If the Side of a Square be 1, the Diameter of a Circle that will circumscribe that Square, will be 1.4142. Therefore,

As 1 : 1.4142 :: 15.98

15.98

113136

127278

70710

14142

22.598916 the Diameter sought.

By Scale and Compasses.

Extend the Compasses from 1 to 1.4142, and that Extent will reach from 15.98 to 22.6, the Diameter sought.

Problem

Problem 14. Having the Side of a Square, to find the Diameter of a Circle equal.

If the Side of a Square be 1, the Diameter of a Circle equal thereunto will be 1.128. Therefore,

Side. Diam. Side of a Side.

As 1 : 1.128 :: 20.0291

1.128

1602328

400582

200291

200291

22.5928248 Diameter.

By Scale and Compasses.

Extend the Compasses from 1 to 1.128; that Extent will reach from 20.0291 (the Side of the Square given) to 22.6, the Diameter of the Circle sought.

Problem 15. Having the Side of a Square, to find the Circumference of the circumscribing Circle.

If the Side of a Square be 1, the Circumference of a Circle that will encompass that Square, will be 4.443. Therefore,

Side Sq. Circum. Side Sq.

As 1 : 4.443 :: 15.98

15.98

35544

39987

22215

4443

70.99914 the Circumf.

By Scale and Compasses.

Extend the Compasses from 1 to 4.443; that Extent will reach from 15.98 to 71, the Circumference.

Problem

Problem 16. Having the Side of a Square, to find the Circumference of a Circle that will be equal thereunto.

If the Side of the Square be 1, the Circumference of a Circle that will be equal thereunto; shall be 3.545. Then,

As 1 : 3.545 :: 20.0291

3.545
1001455
801164
1041455
600873

71.0031595 the Circumf.

By Scale and Compasses.

Extend the Compasses from 1 to 3.545; that Extent will reach from 20.0291 to 71, the Circumference sought.

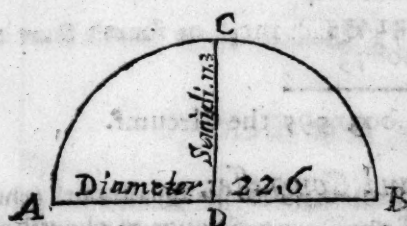
In several of the foregoing Problems, where the Diameter and Circumference is requir'd, the Answers are not exactly the same as the Diameter and Circumference of the given Circle, but are sometimes too much, and sometimes too little, as in the two last Problems, where the Answers in each should be 71, the one being too much, and the other too little. The Reason of this is the small Defect that happens to be in the decimal Fractions, they being sometimes too great, and sometimes too little; yet that Defect is so small, that it is needless to calculate them to more Exactness.

§ X. Of a SEMICIRCLE.

TO find the Area of a Semicircle, this is

The RULE.

Multiply the fourth Part of the Circumference of the whole Circle (that is, half the Arch-Line) by the Semidiameter, the Product is the Area.



Let ABC be a Semicircle, whose Diameter is 22.6, and the half Circumference, or Arch-Line, ACB, is 35.5; the half thereof is 17.75, which multiply by the Semidiameter 11.3, and the Product is 200.575, the Area of the Semicircle.

17.75 the half Arch-Line.

11.3 the Semidiameter.

5325

1775

1775

200.575 The Area of the Semicircle.

By Scale and Compasses.

Extend the Compasses from 1 to 11.3; that Extent will reach from 17.75 to 200.575, the Area.

If only the Diameter of the Semicircle be given, you may say, by the *Rule of Three*,

As 1 is to .3927, so is the Square of the Diameter to the Area.

By

By Scale and Compasses.

Extend the Compasses from 1 to the Diameter 22.6; that Extent turn'd twice from .3927, will reach, at the last, to 200.575.



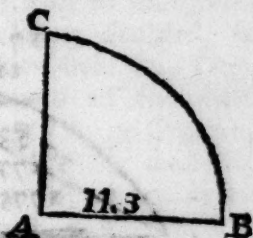
§ XI. Of a QUADRANT.

TO find the Area of a Quadrant, or fourth Part of a Circle, this is

The RULE.

Multiply half the Arch-line of the Quadrant (that is, the eighth Part of the Circumference of the whole Circle) by the Semidiameter, and the Product is the Area of the Quadrant.

Let ABC be a Quadrant, or fourth Part of a Circle, whose Radius, or Semidiameter, is 11.3, and the half Arch-line 8.875; these multiply'd together, the Product is 100.2875 for the Area.

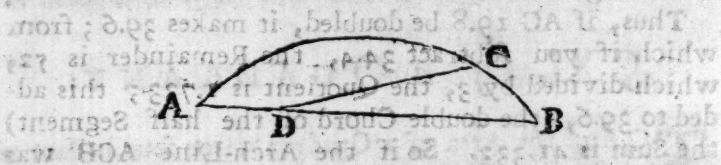


These are the Rules and Ways commonly given for finding the Area of a Semicircle and Quadrant; but, I think, it is as good a Way, to find the Area of the whole Circle, and then take half that Area for the Semicircle, and a fourth Part for the Quadrant.

Before I proceed to shew how to find the Area of the Sector, and Segment of a Circle, I shall shew how to find the Length of the Arch-line, both Geometrically and Arithmetically.

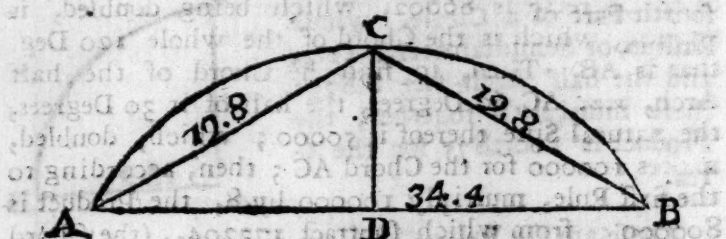
To find the Length of the Arch-line Geometrically.

Divide the Chord-line AB into four equal Parts, and set one of those Parts from B to C, and draw a Line from C to three of those Parts at D; so shall CD be equal to half the Arch-line ACB.



To find the Length of the Arch-line Arithmetically.

Multiply the Chord of half the Segment AC or CB by 8, and from the Product subtract the Chord of the whole Segment AB, and divide the Remainder by 3, the Quotient is the Arch-line ACB sought.



$$\begin{array}{r} 19.8 \text{ AC} \\ 8 \\ \hline \end{array}$$

$$158.4$$

$$34.4 \text{ AB}$$

$$\begin{array}{r} 3 \overline{)124} \\ \hline \end{array}$$

$$\text{Arch-line } 41.333$$

Another

Another Way.

From the double Chord of half the Segment's Arch, subtract the Chord of the Segment, one third Part of the Difference added to the double Chord of half the Segment's Arch, the Sum is the Arch-line of the whole Segment.

Thus, if AC 19.8 be doubled, it makes 39.6; from which if you subtract 34.4, the Remainder is 5.2 which divided by 3, the Quotient is 1.733; this added to 39.6, (the double Chord of the half Segment) the Sum is 41.333. So if the Arch-Line ACB was stretch'd out strait, it would then contain 41.333 such Parts as the Chord AB contains 34.4 of the like Parts.

These two Rules may very easily be prov'd out of the Table of natural Sines; thus,

Suppose (in the former Figure) the Arch ACB to contain 120 Degrees, the natural Sine of half, viz. of 60 Degrees, is 86602, which being doubled, is 173204, which is the Chord of the whole 120 Deg. that is AB. Then, to find the Chord of the half Arch, viz. AC 60 Degrees, the half of it 30 Degrees, the natural Sine thereof is 50000; which, doubled, makes 100000 for the Chord AC; then, according to the first Rule, multiply 100000 by 8, the Product is 800000; from which subtract 173204, (the Chord AB) and the Remainder is 626796; which divide by 3, the Quotient is 208932, which is the Length of the Arch-line ACB, according to the first Rule. Now let us examine how near this comes to the true Quantity of the Arch propos'd: If the Radius or Semidiameter of a Circle be 100000, (as it is in the Table of Sines) then the Circumference will be 628318; and because 120 Degrees is the third Part of the Circle, take the third Part of 628318 is 209439, which is the true Quantity of the Arch ACB in such Parts as

the Semidiameter contains 100000, and differs from that before found 507, which is a Thing inconsiderable in *Practical Mensuration*. The latter of the foregoing Rules agrees exactly with the former, and therefore the Difference will be the same as above; either of the Rules gives the Quantity of the Arch-line too little, and the greater the Arch, the greater the Error. If you know the Degrees that are contain'd in the Segment's Arch, and would have the Arch-line very exactly, you may reason thus by the *Rule of Three*.

As the Circle's Periphery in Degrees is to its Periphery in equal Parts :: so is the Arch in Degrees and decimal Parts : to the same Arch in equal Parts.

Suppose the Circumference of a Circle be 71, and suppose the Arch to contain 52 Degrees, 15 Minutes, (the Decimal of 15 Minutes is .25) then say,

Deg. Parts. Deg.

As 360 : 71 :: 52.25

71

5225

36575

361037019.75 (10.305 fere.

36

109

108

175

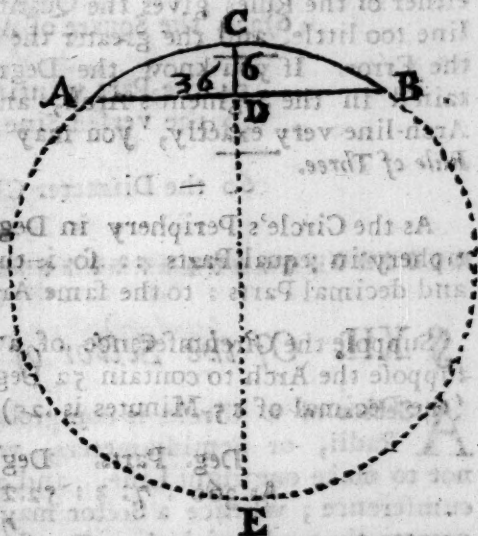
So the 52 Degrees 15 Minutes will contain 10.305 of such Parts as the Circumference contains 71.

Thus have I shew'd several Ways of finding the Measure of the Curve-line of any Part of a Circle very near the Truth. The next Thing I shall shew, is,

How

How to find the Diameter of a Circle, by having the Chord and versed Sine of the Segment Arithmetically.

Because the Chord AB cuts the Diameter EC at right Angles, therefore the Semichord AD or DB is a mean proportional Line between the Parts of the Diameter CD and DE, (by *Eucl. 6.13.*) Therefore, if you square the Semichord AD or DB, and divide that Square by the versed Sine CD, the Quotient will be the Part of the Diameter wanting; to which add the given versed Sine CD, and the Sum is the Diameter sought.



Example. Let ACB be a Segment given, whose Chord AB is 36, and the versed Sine CD 6; half 36 is 18, which squared, makes 324; this divided by 6, the Quotient is 54; to which add 6; the Sum is 60, the Diameter of the Circle CF.

18 half the Chord.

18

144

18

6)324 the Square of AD.

54 the Part wanting DE.

6 the verfed Sine CD add

60 the Diameter CE.

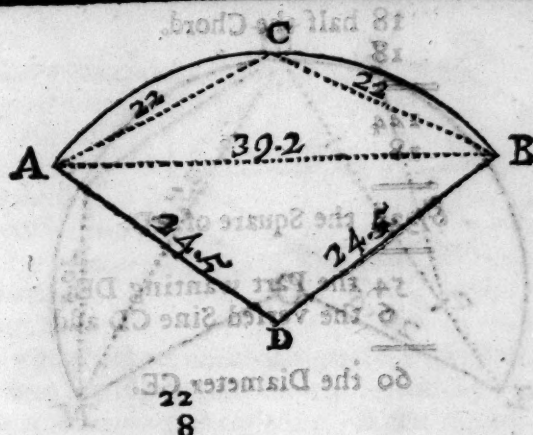
§ XII. Of the Sector of a CIRCLE.

A Sector of a Circle is comprehended under two Radii, or Semidiameters, which are suppos'd not to make one right Line, and a Part of the Circumference; whence a Sector may be either less or greater than a Semicircle. To find the Area or superficial Content thereof, this is

The R U L E.

Multiply half the Arch-line by the Semidiameter, and the Product is the Area.

Let ADBC be the Sector of a Circle given, whose Semidiameter AD or DB is 24.5, and the Arch-line ACB (by the first Rule, pag. 110) I find to be 45.6, the half thereof, 22.8, being multiply'd by 24.5, (the Semidiameter) the Product is 558.6, which is the Area of the Sector ACBD.



176

39.2 Subtrahend.

3)136.8

45.6 Arch-line.

22.8 half.

24.5 Semidiameter.

1140

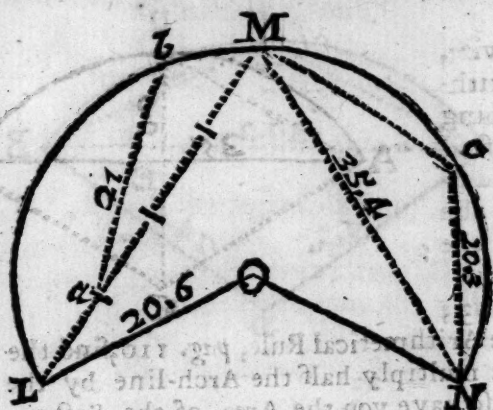
912

456

558.60 the Area.

Again. Let LMNO be a Sector greater than a Semicircle, whose Semidiameter LO or NO is 20.6, and the Line *ba* equal to a fourth Part of the Arch-line LbMN 21, the Double whereof is 42, equal to the Arch-line LbM or McN; or by the Arithmetical Rule, pag. 110, the said Arch is found to be 42.333, which multiply'd by 20.6, the Semidiameter makes 872.0598 for the Area of the Sector LMNO.

See the following Work.



20.3 Chord Nr.
8

42.333
20.6

1624

253998

35.4 Chord MN.

84666

3)127.0

872.0598 Area.

42.333 Arch-line.

§ XIII. Of the Segment of a CIRCLE.

A Segment of a Circle, is a Part terminated by a Right-line less than the Diameter call'd a Chord, and by a Part of the Circumference.

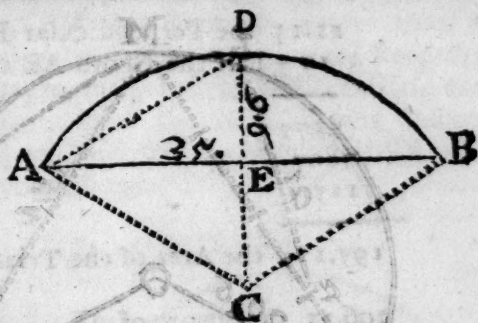
To find the Area of the Segment of a Circle, you must, first, find the Center of the whole Circle, and draw the two Semidiameters, thereby completing the Sector, as in the following Figure. Then (by the last Section) find the Area of the whole Sector CADBC, and then (by Sect. 5) find the Area of the Triangle ABC, and subtract the Area of the Triangle out of the Area of the Sector, the Remainder is the Area of the Segment.

Other-

Otherwise,
you may, with-
out describing
the Figure, find
the Semidia-
meter of the
Circle by the
Arithmetical

Rule, pag. 113;

and by the Arithmetical Rule, pag. 110, find the Arch-
line; then multiply half the Arch-line by the Semi-
diameter, so have you the Area of the Sector. Then
subtract the versed Sine from the Semidiameter, the
Remainder is the Perpendicular of the Triangle; and
multiply the half Chord by the Perpendicular, the
Product is the Area of the Triangle. Then subtract
the Area of the Triangle from the Area of the Sector,
and the Remainder is the Area of the Segment. See
the Work.



$$2)35=AB$$

$$17.5$$

$$17.5$$

$$875$$

$$1225$$

$$175$$

$$9.6)306.25$$

$$(31.9$$

$$182$$

$$865$$

$$9.6 \text{ add}$$

$$41.5 \text{ the Diameter of the Circle.}$$

$$20.75 \text{ the Semidiameter.}$$

$$9.6 \text{ DE Subtrahend.}$$

$$11.15 \text{ remains the Perpendicular EC.}$$

11.15 the Perpendicular EC.

17.7 half the Chord AE or EB.

5575

7803

1115

195.125 the Area of the Triangle.

306.25 the Square of AE.

92.16 the Square of DE the versed Sine.

398.41 Sum.

The square Root thereof is 19.96 the Chord AD.

8

159.68

Sub. 35 the Chord AB.

3)124.68

2)41.56 the Arch-line.

20.78 half.

20.75 Semidiameter.

10390

14546

4156

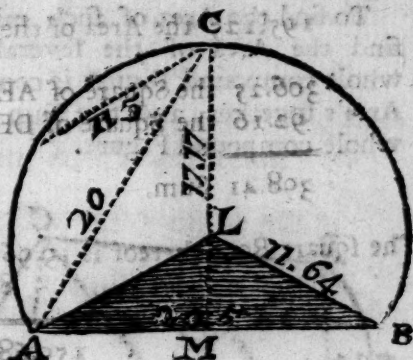
From 431.1850 Area of the Sector,

Subtract 195.125 Area of the Triang.

Remains 236.060 Area of the Segment.

Again,

Again, Let MACBM be a Segment greater than a Semicircle; observe the former Rules in all Respects, as in the last Example, only instead of subtracting the Area of the Triangle out of the Area of the Sector, here you must add it thereunto, as may plainly appear by the following Figure.



11.5
8

92.0
20

3)72

24 half Arch-line.

11.64 Semidiam.

24

4656

2328

279.36 Area of the Sector LACBL.

17.17

11.64

5.53 LM.

10.25 half the Base MA.

5.53 the Perpendicular LM.

3075

5125

5125

56.6825 the Area of the Triangle ALM.

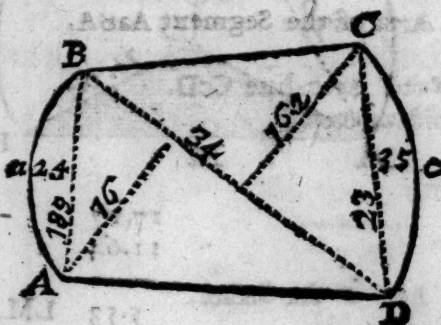
279.36 the Area of the Sector add.

336.0425 the Area of the Segment sought.

S XIV. Of Compound Figures.

Mix'd or compound Figures, are such as are compos'd of rectilineal and curvilineal Figures together.

To find the Area of such mix'd Figures, you must find the Area of the several Figures of which the whole compound Figure is compos'd, and add all the Area's together, and the Sum will be the Area of the whole compound Figure.



16.2 } add.
16.

32.2 Sum.

16.1 half.

34 Diagonal

644

483

547.4 Area of the
(Trapezium.

10.236 half the Arch-line AaB.

14.83 Semidiameter of the Arch AaB.

30708

81888

40944

10236

151.79988 Area of the Sector.

From 14.83 Semidiameter,
 subtract 3.4 versed Sine.

Rem. 11.43 Perpend. of the Triangle.
 9.45 half the Chord AB.

5715
 4572
 10287

108.0135 the Area of the Triangle subtracted
 151.7999 from the Area of the Sector.

43.7864 the Area of the Segment AaBA.

12.19 half the Arch-line CcD.
 20.64 Semidiameter.

4876
 7314
 2438

251.6016 the Area of the Sector.

From 26.64 the Semidiameter,
 subtract 3.5 versed Sine.

Remaind. 17.14 Perpendicular of the Triangle.
 11.5 half the Chord DC.

8570
 1714
 1714

Subtr. 197.110 Area of the Triangle subtracted
 from 251.602 the Area of the Sector.

Rem. 54.492 the Area of the Segment CcDC.
 43.786 the Area of the Segment AaBA.
 547.4 the Area of the Trapezium.

Sum 645.678 the Area of the whole.

M

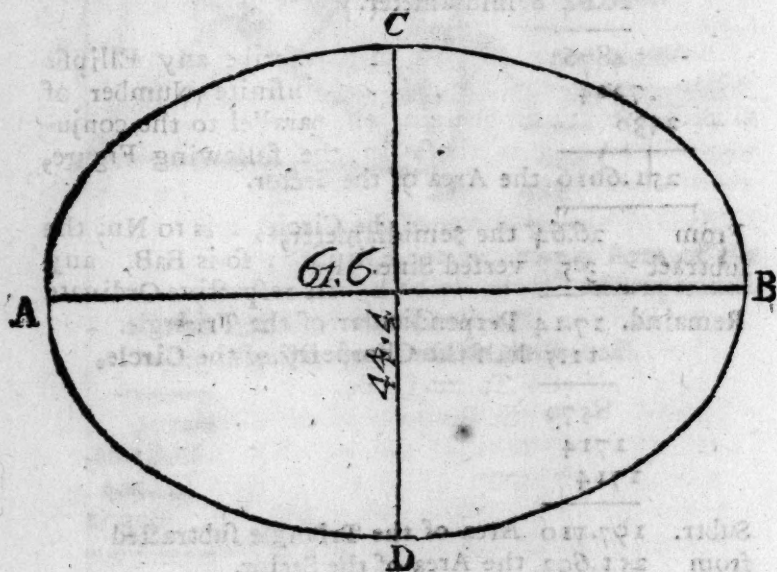
§ XV.

§ XV. Of an ELLIPSIS.

AN Ellipsis, or Oval, is a Figure bounded by a regular Curve-line, returning into it self; but of its two Diameters, cutting each other in the Center, one is longer than the other, in which it differs from the Circle. To find the Area thereof, this is

The RULE.

Multiply the transverse Diameter by the conjugate, and multiply that Product by .7854, this last Product is the Area of the Oval.



61.6

61.6 the transverse Diameter.

44.4 the conjugate Diameter.

2464

2464

2464

2735.04 the Rectangle.

.7854 the Area of Unity.

1094016

1267520

2188032

1914528

2148.100416 the Area of the Oval.

Demonstration. If you circumscribe any Ellipsis with a Circle, and suppose an infinite Number of Chord-lines drawn therein, all parallel to the conjugate Diameter, as those in the following Figure, then it will be,

As DA, the Diameter of the Circle, : is to Nn, the conjugate Diameter of the Ellipsis :: so is BaB, any Chord in the Circle, to ba b, its respective Ordinate in the Ellipsis.

For, according to the Property of the Circle,

it is | 1 | $aS \times Ta = \square Ba.$

And | by the Property of the Ellipsis,

it is | 2 | $\square TC : \square NC :: aS \times Ta : \square ba.$

1, 2, | 3 | $\square TC : \square NC :: \square Ba : \square ba.$

3, hence | 4 | $TC : NC :: Ba : ba.$

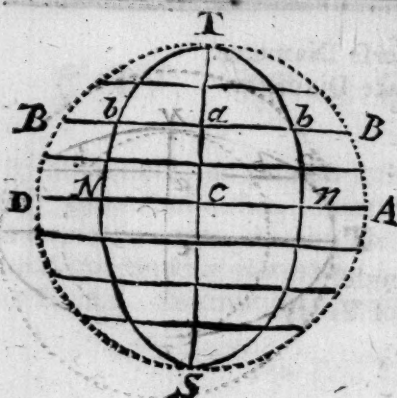
Conseq. | 5 | $2TC : 2NC :: 2Ba : 2ba.$

That is | 6 | $DA : Nn :: BaB : bab.$

But the Sum of an infinite Series of such Chords, as BaB, do constitute the Area of the Circle. And the Sum of the like Series of their respective Ordinates, as ba b, do constitute the Area of the Ellipsis.

M 2

Therefore,



Therefore, As TS :
to Nn :: Circles A-
rea : to the Ellipsis
Area. But $TS : Nn$
:: $\square TS : \square Nn$;
whence it follows
that,

As $\square TS$: Circles
Area :: so is $\square Nn$:
Ellipsis Area.

Consequently, As 1 : to $.7854$:: so is the Rect-
angle, or Product of the transverse and conjugate Di-
ameter of any Ellipsis : to its Area.

Hence it is easy to conceive, that the Square Root
of the Product of the transverse and conjugate Diame-
ters, will be the Diameter of a Circle equal to the
Ellipsis.

Hence also all Segments of an Ellipsis, and its cir-
cumscribing Circle, (whose Bases are parallel to the
conjugate Diameter, and of the same Height) are in
Proportion one to another, as their Bases are, that is,

As BaB : bab :: Area Segment BTB : Area Seg-
ment bTb .

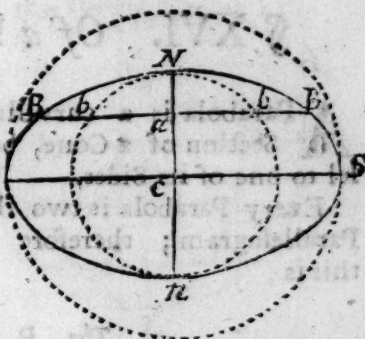
Or, As TS : Nn :: Area Segment BTB : Area Seg-
ment bTb .

The Area of every Ellipsis is a mean Proportional
between the Area's of its circumscribing and inscrib'd
Circles.

The

The Truth of this may easily be deduc'd from the last; for it is already prov'd, that
 $\square TS : TS \times Nn ::$
 circumscribing Circles
 Area : Ellipsis Area.

But $\square TS : TS \times Nn ::$
 $\square Nn$. Therefore El-
 lipsis Area : inscrib'd
 Circles Area :: $TS \times$
 $Nn : \square Nn$.



Example. Let $TS=36$, and $Nn=18.4$.
 Then $\square TS=1296$, and $\square Nn=338.56$.
 Then $1296 \times .7854 = 1017.8784$ great Circles Area;
 And $338.56 \times .7854 = 265.905$, &c. lesser Circles Area;
 And $36 \times 18.4 = 662.4 \times .7854 = 520.24896$, which is
 the Area of the Ellipsis; then it will be,
 As $1017.878 : 520.24896 :: 520.24896 :$
 265.905024 .

That is, As the great Circles Area : is to the Area
 of the Ellipsis :: so is the Area of the Ellipsis to the
 Area of the lesser Circle.

From hence it follows, that all Segments of an El-
 lipsis, and its inscrib'd Circle, (whose Bases are paral-
 lel to the transverse Diameter, and have the same
 Height) are in Proportion one to another, as the Area
 of the Circle are.

That is, As the Area of the Circle : to the Area of
 the Ellipsis :: so is the Segment bNb : to the Seg-
 ment BNB.

Or, $Nn : TS ::$ Area Segment bNb : Area Seg-
 ment BNB.

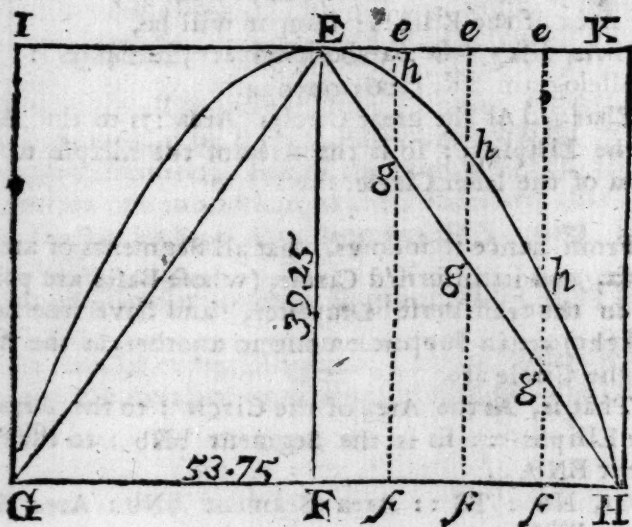
§ XVI. Of a PARABOLA.

A Parabola is a curvilinear Figure, made by the Section of a Cone, being cut by a Plane parallel to one of its Sides.

Every Parabola is two thirds of its circumscribing Parallelogram; therefore to find the Area thereof, this is

The RULE.

Multiply the Base, or greatest Ordinate, by the perpendicular Height, and multiply that Product by 2, and divide the last Product by 3, the Quotient will be the Area of the Parabola.



53.75 the Ordinate GH.
39.25 the Perpendicular EF.

26875
10750
48375
16125
2109.6875
2

3)4219.3750

1406.4583 the Area.

Demonstration. Let FH, the Semi-ordinate, be divided into four equal Parts, or into 8.16, &c. and through the Divisions draw Lines, as ef, ef, &c. parallel to the Axis EF. Suppose also EF to be 4.

Then, I say, the parabolick Space EhHF is to the Parallelogram EKFH as 2 to 3; but to the Triangle EFH as 4 to 3.

For, first, gf, gf, gf, &c. are in continual arithmetical Proportion from the Nature of plain Triangles.

Secondly, fe : ge :: ge : he; but he in the Axis EF=0, and in the first Parallel ef must be equal to $\frac{1}{4}$, in the next ef must be equal to $\frac{2}{4}$, in the third to $\frac{3}{4}$, and so on, in a duplicate arithmetical Progression.

For as ef (=4) : ge (=1) :: so is ge (=1) : to eh ($=\frac{1}{4}$). And as the second ef (=4) to eg (=2) :: so is eg (=2) : to eh ($=\frac{2}{4}$) &c. And thus it will be, if the Lines Ff, ff, &c. be again bisected, &c. *ad infinitum*. So that all the Indivisibles of the trilinear Space EKHhE will be in a duplicate arithmetical Progression increasing. But the Sum of a Rank of such Terms, is subtriple to a Rank of as many equal to the greatest, (by Lemma 3); wherefore the whole trilinear Space EKHhE is to the Parallelogram as 1 to 3; and, consequently, the remaining parabolick Space

Space must be to it as 2 to 3; which was, &c.

And since the Triangle FEH is to the Parallelogram as 1 to 2, it must be to the Parabola as $1\frac{1}{2}$ to 2, or as 3 to 4; which was to be prov'd.

Before I proceed to the Mensuration of solid Bodies, I will lay down such Lemma's as will be necessary to facilitate the Demonstration of all such Solids.

LEMMA I.

In any Series of equal Numbers, (representing Lines or other Quantities) as 1, 1, 1, 1, &c. or 2, 2, 2, 2, &c. or 3, 3, 3, 3, &c. if one of the Terms be multiply'd into the Number of Terms, the Product will be the Sum of all the Terms in the Series.

LEMMA II.

If a Series of Numbers in arithmetical Progressions, begin with a Cypher, and the common Difference be 1, as 0, 1, 2, 3, &c. (representing a Series of Lines or Roots beginning with a Point) if the last Term be multiply'd into the Number of Terms, the Product will be double the Sum of all the Series.

That is, putting L = the last Term, N = the Number of Terms, and S = the Sum of all the Series: Then will $NL = 2S$; consequently, $\frac{1}{2}NL = S$, viz. one half of so many times the greatest Term as there are Number of Terms in the Series.

$$\text{Thus } \left\{ \begin{array}{l} 0+1+2+3+4 = 10 \text{ the Sum} = \frac{1}{2} NL. \\ 4+4+4+4+4 = 20 = NL. \end{array} \right.$$

LEMMA III.

If a Series of Squares, whose Sides or Roots are in arithmetical Progression, beginning with a Cypher, &c. be infinitely continu'd, the last Term, being multiply'd into the Number of Terms, will be triple

to

to the Sum of all the Series, viz. $NLL=3S$; or, $\frac{1}{3} NLL=S$.

That is, the Sum of such a Series will be one third of the last, or greatest Term, so many times repeated as there are Number of Terms in the Series.

Instances in square Numbers.

$$1 \left\{ \frac{0+1+4}{4+4+4} = \frac{1}{12} = \frac{1}{3} + \frac{1}{12} \right.$$

$$2 \left\{ \frac{0+1+4+9}{9+9+9+9} = \frac{14}{36} = \frac{7}{18} = \frac{1}{3} + \frac{1}{18} \right.$$

$$3 \left\{ \frac{0+1+4+9+16}{16+16+16+16+16} = \frac{30}{80} = \frac{3}{8} = \frac{1}{4} = \frac{1}{3} + \frac{1}{12}, \&c. \right.$$

From these Instances it is evident, that as the Number of Terms in the Series do increase, the Fraction, or Excess above $\frac{1}{3}$ does decrease, the said Excess always being $\frac{1}{6N-6}$; which, if we suppose the Series to be infinitely continu'd, will quite vanish, and become nothing at all.

LEMMA IV.

If a Series of Cubes, whose Roots are, in arithmetical Progression, beginning with a Cypher, &c. (as above) be infinitely continu'd, the Sum of all the Series will be $\frac{1}{4} NLLL=S$.

That is, one fourth of the last Term, so many Times repeated as there are Numbers of Terms.

Instances

Instances in Cube Numbers.

If 0, 1, 2, 3, 4, 5, &c. be the Roots of the Cubes,

$$1 \left\{ \frac{0 + 1 + 8 + 27}{27 + 27 + 27 + 27} = \frac{36}{108} = \frac{1}{3} = \frac{1}{4} + \frac{1}{12} \right.$$

$$2 \left\{ \frac{0 + 1 + 8 + 27 + 64}{64 + 64 + 64 + 64 + 64} = \frac{100}{320} = \frac{1}{3.2} = \frac{1}{5} = \frac{1}{4} \right. \\ \left. \left(+ \frac{1}{20} \right) \right.$$

$$3 \left\{ \frac{0 + 1 + 8 + 27 + 64 + 125}{125 + 125 + 125 + 125 + 125 + 125} = \frac{225}{750} = \frac{3}{10} \right. \\ \left. \left(= \frac{1}{4} + \frac{1}{20} \right) \right.$$

From these Examples it plainly appears, that as the Number of Terms in the Series increases, the Fraction or Excess above $\frac{1}{4}$ decreases, the Excess being always $\frac{1}{4N-4}$; which, if we suppose the Series to be infinitely continu'd, will become infinitely small, or nothing.

L E M M A V.

If a Series of Biquadrats, whose Roots are, in arithmetical Progression, beginning with a Cypher, &c. as before, be infinitely continu'd, the Sum of all the Terms in such a Series, will be $\frac{1}{5}$ NLLLL.

The Truth of this may be manifest by the like Process, as in the foregoing Lemma's, and so on for higher Powers.

L E M M A VI.

The Sum of an infinite Progression, whose greatest Term is a square Number, the others decreasing by odd Numbers, viz. 1, 3, 5, &c. is in subsesquialteran Proportion of the Sum of the like Number of equal Terms, that is, as 2 to 3.

Instances

Instances in such Progressions.

$$1 \left\{ \frac{9+8+5}{9+9+9} = \frac{22}{27} = \frac{2}{3} + \frac{4}{9} \right.$$

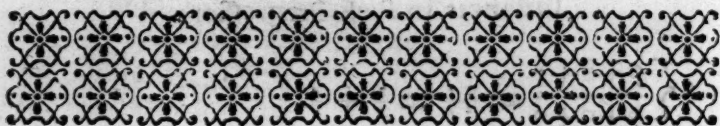
$$2 \left\{ \frac{16+15+12+7}{16+16+16+16} = \frac{50}{64} = \frac{2}{3} + \frac{1}{32} \right.$$

$$3 \left\{ \frac{25+24+21+16+9}{25+25+25+25+25} = \frac{95}{125} = \frac{2}{3} + \frac{1}{37} \right.$$

$$4 \left\{ \frac{36+35+32+27+20+11}{36+36+36+36+36+36} = \frac{161}{216} = \frac{2}{3} + \frac{1}{72} \right.$$

From these Examples it plainly appears, that as the Number of Terms in the Series increases, the Fraction or Excess above $\frac{2}{3}$ decreases; and if we suppose the Series to be infinitely continu'd, that Excess will quite vanish, and the Sum of the infinite Series will be $\frac{2}{3}$ of so many equal to the greatest.





C H A P. II.

The Mensuration of SOLIDS.

SOLID Bodies are such as do consist of Length, Breadth, and Thickness; as Stone, Timber, Globes, Bullets, &c.



§ I. Of a C U B E.

A Cube is a square Solid, comprehended under six geometrical Squares, being in the Form of a Dye. To find the solid Content, this is

The R U L E.

Multiply the Side of the Cube into it self, and that Product again by the Side; the last Product will be the Solidity, or solid Content of the Cube.

17.5

17.5

—
875

1225

175

—
306.25

17.5

—
53125

214375

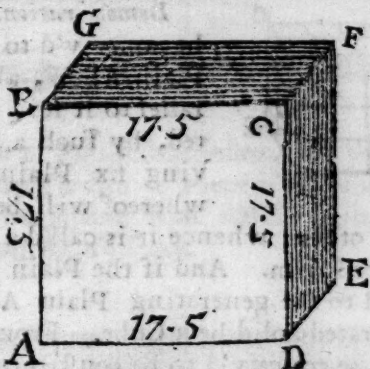
30625

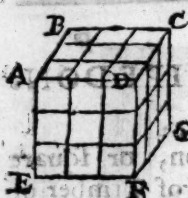
5359.375 the solid Content of the Cube.

Suppose ABCDEFG a cubical Piece of Stone or Wood, each Side thereof being 17 Inches and an half; multiply 17.5 by 17.5, and the Product is 306.25; which being multiply'd by 17.5, the last Product is 5359.375, which is 5359 solid Inches, and 375 Parts. For reduce the solid Inches to Feet, divide by 1728, (because so many cubical Inches is a Foot) and the solid Feet in the Cube will be 3, and 175 cubical Inches remain.

By Scale and Compasses.

Extend the Compasses from 1 to 17.5; that Extent, turn'd over twice from 17.5, will reach to 5359, the solid Content in Inches. Then extend the Compasses from 1728 to 1; that Extent, turn'd the same Way from 5359, will reach to 3.1 Feet.



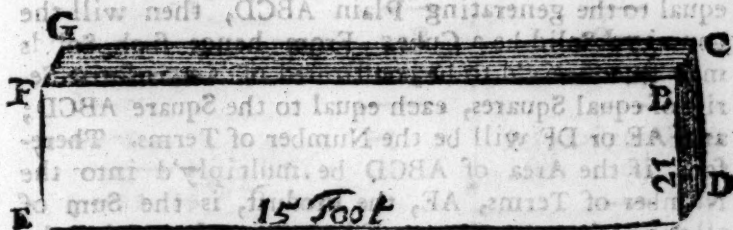


Demonstration. If the Square ABCD be conceiv'd to be mov'd down the Plain ADEF, always remaining parallel to it self, there will be generated, by such a Motion, a Solid having six Plains, the two opposite whereof will be equal and parallel to each other; whence it is call'd a Parallelopipedon, or Square Prism. And if the Plain ADEF be a Square equal to the generating Plain ABCD, then will the generated Solid be a Cube. From hence such Solids may be conceiv'd to be constituted of an infinite Series of equal Squares, each equal to the Square ABCD; and AE or DF will be the Number of Terms. Therefore, if the Area of ABCD be multiply'd into the Number of Terms, AE, the Product, is the Sum of all the Series, (*per* Lemma I.) and, consequently, the Solidity of the Parallelopipedon or Cube. Or if the Base ABCD, being divided into little square Area's, be multiply'd into the Height AE, divided by a like Measure for Length, after this Way you may conceive as many little Cubes to be generated in the whole Solid, as is the Number of the little Area's of the Base multiply'd by the Number of Divisions the Side AE contains. Thus, if the Side of the Base AB be 3, that multiply'd into it self is 9, which is the Area of the square Base ABGD; then, if AE be likewise 3, multiply 9 by 3, and the Product is 27; and so many little Cubes will this Solid be cut into, if you conceive it to be cut as the Lines direct.

From this Demonstration it is very plain, that if you multiply the Area of the Base of any Parallelopipedon into its Length or Height, that Product will be the solid Content of such a Solid.

§ II. Of a PARALLELOPIPEDON.

LET ABCDEFG be a ParallelopiPEDON, or square Prism, representing a square Piece of Timber or Stone, each Side of its square Base ABCD being 21 Inches, and its Length AE 15 Foot.



First, then, multiply 21 by 21, the Product is 441, the Area of the Base in Inches; which multiply'd by 180, the Length in Inches, and the Product is 79380, the solid Content in Inches: Divide the last Product by 1728, and the Quotient is 45.9, that is, 45 solid Feet, and 9 Tenths of a Foot. Or thus: Multiply 441 by 15 Feet, and the Product is 6615; divide this by 144, and the Quotient is 45.9, the same as before.

Or thus, by multiplying Feet and Inches.

Multiply 1 Foot 9 Inches by 1 Foot 9 Inches, and the Product is 3 Feet, 0 Inches, 9 Parts; this multiply'd again by 15 Feet, gives 45 Feet, 11 Inches, 3 Parts, that is, 45 Feet, and $\frac{1}{2}$ of a Foot, and $\frac{1}{4}$ of 1.

See the Work of all these.

21	441	F, 1
—	15	1-9
21	—	1-9
42	2205	1-9
—	441	1-3-9
441	—	—
180	1446615(45.9	3-0-9
—	—	15
35280	855	—
441	1350	45-0-0
—	—	7-6
1728)79380(45.9	54	3-9
6912	—	45-11-3
—	—	—
10260	—	—
8640	—	—
—	—	—
16200	—	—
15552	—	—
—	—	—
648	—	—

By Scale and Compasses.

Extend the Compasses from 12 to 21, and that Extent will reach to near 46 Foot, being twice turn'd over from 15 Foot; so the solid Content is almost 46 Feet.

If the Base of the squar'd Solid be not an exact Square, but in Form of a rectangle Parallelogram, the Way of measuring of it is much the same; for, first, you must find the Area of the Base, by multiplying the Breadth by the Depth; and then multiply that Area by the Length of the Piece, as before: Thus,

If a Piece of Timber be 25 Inches broad, 9 Inches deep, and 25 Feet long, how many solid Feet are contain'd therein?

25	F. I.
9	2—1
—	0—9
225	—
25	1—6—9
—	25
1125	—
450	25—0—0
—	12—6—0
144)5625(39	1—0—6
432	0—6—3
—	—
1305	39—0—9
1296	—
—	—
09	—

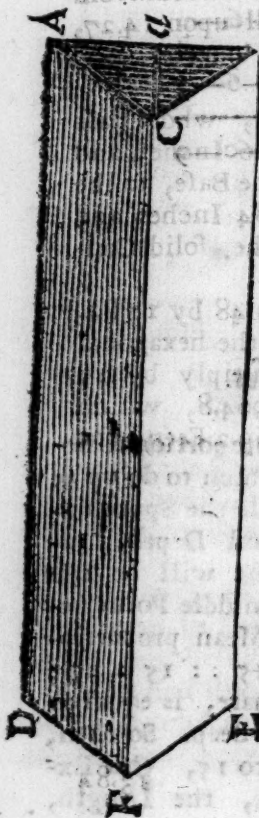
Answer 39 Feet.

By Scale and Compasses.

First, find a Mean geometrical, proportional between the Breadth and the Depth; which to do upon the Line of Numbers, you must divide the Space upon the Line, between the Breadth and Depth, into two equal Parts, that middle Point will be the Mean proportional sought; thus the middle Point between 25 and 9 is at 15; so is 15 a Mean proportional between 9 and 25; for, as 9 : 15 :: 15 : 25; so a Piece of Timber of 15 Inches square, is equal to a Piece 25 Inches broad and 9 Inches deep. So then, if you extend the Compasses from 12 to 15, that Extent, turn'd twice over from 25 Feet, the Length, will reach to 39 Feet, the Content.

§ III. Of a Triangular PRISM.

A Prism is a Solid contain'd under several Plains, and having its Bases like, equal, and parallel: The solid Content of a Prism (whether triangular or multangular) is found by multiplying the Area of the Base into the Length or Height, and the Product is the solid Content.



Let ABCDEF. be a triangular Prism, each Side of the Base being 15.6 Inches, the Perpendicular thereof Ca is 13.51 Inches, and the Length of the Solid 19.5 Feet.

Multiply the Perpendicular of the Triangle 13.51 by half the Side 7.8, and the Product is 105.378, the Area of the Base; which multiply by the Length 19.5, and the Product is 2054.871, which divide by 144, and the Quotient is 14.27 Feet, *ferè*, the solid Content.

13.51	144	2054.87	14.27
7.8	144		
10808	614		
9457	576		
105.378	388		
19.5	288		
526890	1007		
948402	1008		
105378			

2054.8710

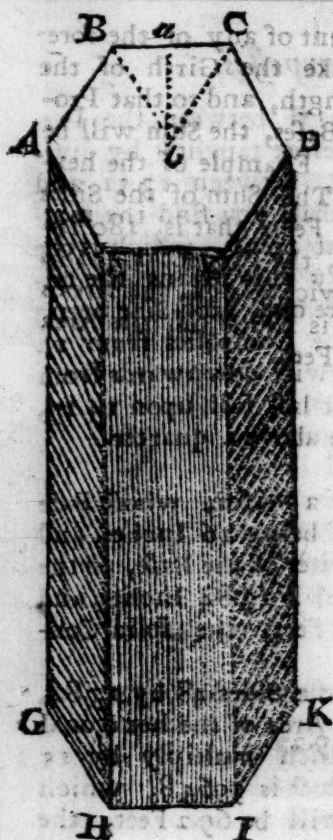
By

By Scale and Compasses.

First, find a Mean proportional between the Perpendicular and half Side, (as before taught) by dividing the Space upon the Line, between 13.51 and 7.8, into two equal Parts; so shall you find the middle Point between them to be at 10.26, which is the Mean proportional sought: By this Means the triangular Solid is brought a square one, each Side being 10.26 Inches. Then extend the Compasses from 12 to 10.26; that Extent, turn'd twice downwards from 19.5 Feet, the Length, will at last fall upon 14.27, which is 14 Feet, and a little above a quarter.

Let ABCDEFGHIK represent a Prism, whose Base is a Hexagon, each Side thereof being 16 Inches, and the Perpendicular, from the Center of the Base, to the Middle of one of the Sides, (a b) is 13.84 Inches, and the Length of the Prism is 15 Feet, the solid Content is requir'd.

Multiply half the Sum of the Sides 48 by 13.84, and the Product is 664.32, the Area of the hexagonal Base, (by § VIII, P. 86) which multiply by 15 Feet, the Length, and the Product is 9964.8, which divided by 144, the Quotient will be 69.2 Feet, the solid Content requir'd.



$$\begin{array}{r}
 13.84 \\
 48 \\
 \hline
 11072 \\
 5536 \\
 \hline
 664.32 \text{ Ar. of Base} \\
 15 \\
 \hline
 332160 \\
 66432 \\
 \hline
 1449964.80 (692 \\
 864 \\
 \hline
 1324 \\
 1296 \\
 \hline
 288 \\
 288 \\
 \hline
 \dots
 \end{array}$$

By Scale and Compasses.

First, find a Mean proportional between the Perpendicular, and half the Sum of the Sides, that is, divide the Space between 13.84 and 48, and the middle Point will be 25.77 : Then extend the Compasses from 12 to 25.77 ; that Extent will reach (being twice turn'd over) from 15 Feet, the Length, to 69.2 Feet, the Content.

Extend the Compasses from 144 to 180, that Extent will reach from 96 to 120 Feet. Then, to find the Content of the Base, extend the Compasses from 144

To find the superficial Content of any of the fore-mention'd Solids, you must take the Girth of the Piece, and multiply by the Length, and to that Product add the two Area's of the Bases, the Sum will be the whole superficial Content. Example of the hexagonal Prism last mention'd: The Sum of the Sides being 96, and the Length 15 Feet, that is, 180 Inches, which multiply'd by 96, the Product is 17280 square Inches; to which add twice 664.32, the Area's of the two Bases, and the Sum is 18608.64, the Area of the whole, which is 129.22 Feet.

$$\begin{array}{r}
 180 \\
 96 \\
 \hline
 1080 \\
 1620 \\
 \hline
 17280 \\
 664.32 \\
 664.32 \\
 \hline
 14418608.64(129.22
 \end{array}$$

$$\begin{array}{r}
 420 \\
 1328 \\
 326 \\
 \hline
 384
 \end{array}$$

96

The superficial Content of the whole Solid is 129.22 Feet.

By Scale and Compasses.

Extend the Compasses from 144 to 180, that Extent will reach from 96 to 120 Feet: Then, to find the Area of the Base, extend the Compasses from 144

to

to 13.84, that Extent will reach from 48 to 4.6 Feet; add 120 Feet, and twice 4.6 Feet, and it makes 129.2 Feet, the superficial Content, as before.

The Demonstration of those last Solids, will be the same as in the first Section; for as in that, so in those, the Area of the Base is multiply'd into the Length to find the Content, and the same Reason is given for one as for the other.

§ IV. Of a PYRAMID.

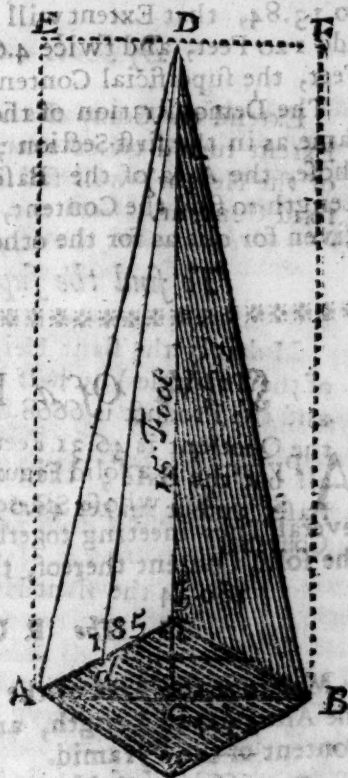
A Pyramid is a solid Figure, whose Base is a Polygon, and whose Sides are plain Triangles, their several Tops meeting together in one Point. To find the solid Content thereof, this is

The RULE.

Multiply the Area of the Base by a third Part of the Altitude, or Length, and the Product is the solid Content of the Pyramid.

Let

Let ABD be a square Pyramid, each Side of the Base being 18.5 Inches, and the perpendicular Height CD is 15 Feet: Multiply 18.5 by 18.5, and the Product is 342.25, the Area of the Base in Inches; which multiply'd by 5, a third Part of the Height, and the Product is 1711.25; this divided by 144, the Quotient is 11.88 Feet, the solid Content.



18.5
18.5

925
1480
185

342.25 Area of the Base.
5

144) 1711.25 (11.88 Content.

F. I. Pts.

1 6 6
1 6 6

1 6 6

9 3

9 3

2 4 6 3

5

11 10 7 3

By Scale and Compasses.

Extend the Compasses from 12 to 18.5 Inches, that Extent turn'd twice over from 5 Feet, (a third Part of the Height) will fall at last upon 11.88 Feet, the solid Content.

To find the superficial Content,

Multiply the slant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Base 37, and the Product is 6668.88, which divided by 144, the Quotient is 46.31 Feet, the superficial Content of all but the Base; then to that add 2.38 Feet, the Base, and it makes 48.69 Feet, the whole superficial Content.

180.24 the slant Height d D.

37

126168

54072

144)6668.88(46.31

576

2.38

908

48.69 the whole Content.

864

448

432

168

144

24

144)342.25(2.38

288

542

432

1105

1152

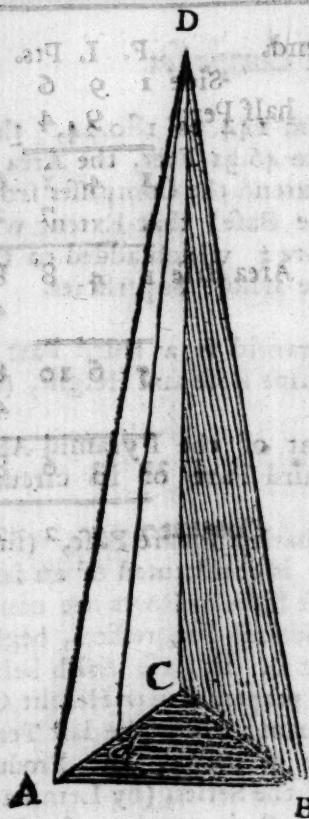
By Scale and Compasses.

Extend the Compasses from 144 to 180.24, that Extent will reach from 37 to 46.31 Feet, the Area of the four Triangles. And extend the Compasses from 144 to 18.5 (one Side of the Base) that Extent will reach from 18.5 to 2.38, *ferè*; which added to the other, the Sum is 48.69, the whole Superficies.

Demonstration. Every Pyramid is a third Part of the Prism, that hath the same Base and Height, (by *Euc. 12, 7.*)

That is, the solid Content of the Pyramid ABD (in the last Figure) is one third Part of its circumscribing Prism ABEF.

For every Pyramid that hath a square Base, (such as AaBb in the last Figure) is constituted of an infinite Series of Squares, whose Sides or Roots are continually increasing in arithmetical Progression, beginning at the Vertex or Point D, its Base AaBb being the greatest Term, and its perpendicular Height CD is the Number of all the Terms: But the last Term multiply'd into the Number of Terms, the Product will be triple the Sum of all the Series, (by Lemma 3) consequently $\frac{N \times L}{3} = S$. And S is equal to the solid Content of the Pyramid. From hence it will be easy to conceive, that every Pyramid is $\frac{1}{3}$ of its circumscribing Prism, (that is, of a Prism of equal Base and Altitude) what Form soever its Base is of, *viz.* whether it be square, triangular, pentangular, &c. You may very easily prove a triangular Pyramid to be a third Part of a Prism of equal Base and Altitude, by cutting a triangular Prism of Cork, and then cut that Prism into three Pyramids, by cutting diagonally, as I have several Times done, to satisfy my self and others.



Let ABCD be a triangular Pyramid, each Side of the Base being 21.5 Inches, and its perpendicular Height 16 Feet, the Content solid and superficial, is requir'd.

First, find the Area of the Base, by multiplying half the Side by the Perpendicular let fall from the Angle of the Base, to the opposite Side; which Perpendic. will be found to be 18.62; the half thereof is 9.31, multiply'd by 21.5, the Product is 200.165 Inches, the Area of the Base. Then, because the Altitude 16 cannot exactly be divided by 3, therefore I take the third Part of 200.165, which is 66.72, and multiply it by 16, and the Product is 1067.52, which divided by 144, the Quotient is 7.41 Feet, the solid Content.

9.31	half the Perpend.	F. I. Pts.	
21.5	the Side.	Side	1 9 6
		half Perp.	9 4
4655			
931			1 4 1 6
1862			7 2
3)200.165	Area Base.	Area Base	1 4 8 8
			4
66.72	a third Part.		
16	Height.		5 6 10 8
			4
40032			
6672			3)22 3 6 8
144)1067.52	(7.41 solid Cont.	Content	7 5 2 2
1008			
595			
576			
192			
144			
48			

In casting this up by Feet and Inches, instead of multiplying by 16, the Height, I break 16 into two such Numbers, as being multiply'd together, the Product may be 16, viz. into 4 and 4, and multiply by first one, and then by the other; a third Part of the last Product is the Content.

By Scale and Compasses.

First, find a geometrical Mean proportional, (as before directed) by dividing the Space between 21.5 and 9.31 into two equal Parts, and you will find the middle Point at 14.15, which is the Mean proportional sought. Then extend the Compasses from 12 to

14.15, that Extent (turn'd twice over from 16 Feet) will fall at last upon 22.23; a third Part thereof is 7.41 Feet, the Content.

To find the superficial Content,

Multiply the slant Height (or Perpendicular of one of the Triangles) by half the Periphery of the Base, and to that Product add the Area of the Base, the Sum is the whole superficial Content.

192.1 Inches, the slant Height &c.
Half Periph. 32.25 = 21.5 + 10.75.

$$\begin{array}{r} 9605 \\ 3842 \\ 3842 \\ \hline 2763 \end{array}$$

6195.225 Inches, the Area of all but the Base.
200.165 Area of the Base add.

144)6395.390 (44.41 Feet, the whole Content.

$$\begin{array}{r} 576 \\ \hline 635 \\ 576 \\ \hline 593 \\ 576 \\ \hline 179 \\ 144 \\ \hline 35 \end{array}$$

By Scale and Compasses.

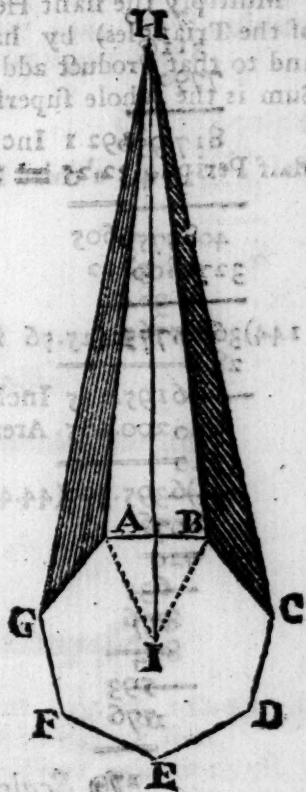
Extend the Compasses from 144 to 192.1, that Extent will reach from 32.25 (half the Periphery of the Base) to 43.02 Feet, the Content of the upper Part.

And

And extend the Compasses from 144 to half the Perpendicular 9.31, that Extent will reach from the Side 21.5 to 1.39 Feet, the Area of the Base; which added to the other, makes 87.57 Feet, the Content of the whole.

Let ABCDEFGH be a Pyramid, whose Base is a Heptagon, each Side thereof being 15 Inches, and the Perpendicular of the Heptagon is 15.58 Inches, and the Perpendicular Height of the Pyramid HI is 13.5 Feet; the Content solid and superficial is requir'd.

Multiply 15.58 (the Perpendicular) by 32.5, (half the Sum of the Sides of the Heptagon) and the Product is 506.39, which multiply'd by 4.5, viz. $\frac{1}{2}$ of the Height, and the Product is 2278.75.



Then

Then divide this last Product by 144, and the Quotient is 25.56 Feet, the Content.

5.58 the Heptagon's Perpendicular.

52.5 the half Sum of the Sides.

7790

3116

7790

817.950

4.5 a third Part of the Height.

4089750

3271800

144)3680.7750(25.56 Solid Feet.

288

800

720

807

720

877

864

13

By Scale and Compasses.

First, find a geometrical Mean proportional between 15.58 and 52.5, (as is before directed) which you will find to be 28.06; then extend the Compasses from 12 to 28.06, that Extent will reach from 4.5 (twice turn'd over) to 25.56 Feet.

To find the Superficial Content,

Multiply the Height taken from the Middle of one of the Sides of the Base 162.75 Inches, by the half Sum of the Sides 52.5 Inches, and the Product is 8544.375; which divided by 144, the Quotient is 59.335 Feet, the Content of the upper Part.

162.75

52.5

81375

32550

81375

144)8544.375(59.335 Feet.

5.68 Base add:

1344

483

517

855

135

65.015 the whole Content.

By Scale and Compasses.

Extend the Compasses from 144 to 162.75, that Extent will reach from 52.5 to 59.335 Feet.

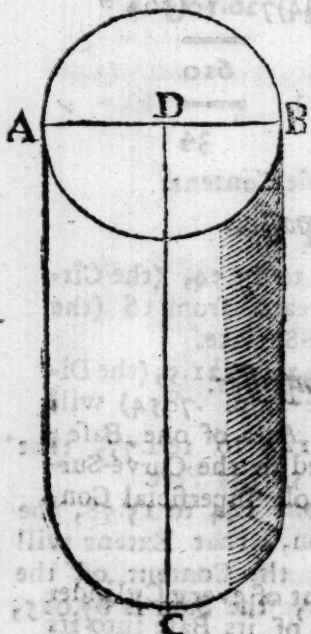
And extend the Compasses from 144 to 15.58, the Perpendicular of the Hepragon, that Extent will reach from 52.5 to 5.68 Feet, the Content of the Base; which add to the former, the Sum is 65.015, the whole superficial Content.

§ V. Of a CYLINDER.

A Cylinder is a round Solid, having its Bases circular, equal, and parallel, in Form of a Rolling-Stone us'd in Gardens. To find the solid Content thereof, this is

The RULE.

Multiply the Area of the Base by the Length, and the Product is the solid Content.



Let ABC be a Cylinder, whose Diameter AB is 21.5 Inches, and the Length CD is 16 Foot, the solid Content is requir'd.

First, square the Diameter 21.5, and it makes 462.25, which multiply by .7854, and the Product is 363.05115. Then multiply this by 16, and the Product is 5808.8164. Divide this last Product by 144, and the Quotient is 40.34 Feet, the solid Content.

By Scale and Compasses.

Extend the Compasses from 12.54 to 21.5, the Diameter, that Extent (turn'd twice over, from 16, the Length) will at last fall upon 40.34, the solid Content.

To

To find the Superficial Content,

First, (by Chap. I. Sect. IX. Prob. 2.) find the Circumference of the Base 67.54, which multiply by 16, the Product is 1080.64; which divided by 12, the Quotient is 90.05 Feet, the Curve-Surface; to which add 5.04 Feet, the Sum of the two Bases, and the Sum is 95.09 Feet, the whole Superficial Content.

$$\begin{array}{r}
 67.54 \\
 \times 16 \\
 \hline
 40524 \\
 6754 \\
 \hline
 12)1080.64 \\
 \hline
 90.05
 \end{array}
 \qquad
 \begin{array}{r}
 90.05 \\
 5.04 \\
 \hline
 95.09 \text{ Sum.}
 \end{array}
 \qquad
 \begin{array}{r}
 363.05 \\
 \times 2 \\
 \hline
 726.10 \\
 144)726.10(5.04 \\
 \hline
 610 \\
 \hline
 34
 \end{array}$$

By Scale and Compasses.

Extend the Compasses from 12 to 67.54, (the Circumference) that Extent will reach from 16 (the Length) to 90.05 Feet, the Curve-Surface.

And extend the Compasses from 12 to 21.5, (the Diameter) that Extent (turn'd twice from .7854) will at last fall upon 2.52 Feet, the Area of one Base; which doubled is 5.04; this added to the Curve-Surface, makes 95.09 Feet, the whole Superficial Content.

Demonstration. The solid Content of every Cylinder is found by multiplying the Area of its Base into its Height, as aforefaid: For every right Cylinder is only a round Prism, being constituted of an infinite Series of equal Circles; that of its Base, or End, being one of the Terms, and its Height CD (in the former Figure) is the Number of all the Terms. Therefore the Area of its Base AB being multiply'd into CD,

CD, will be its Solidity, (by Lemma I.) Let $D = AB$, and $H = CD$.

Then $.7854 DD \div H =$ its Solidity.



§ VI. Of a C O N E.

A Cone is a Solid, having a circular Base, and growing smaller and smaller, till it ends in a Point, which is call'd the Vertex, and may be nearly represented by a Sugar-loaf. To find the Solidity thereof, this is

The R U L E.

Multiply the Area of the Base by a third Part of the perpendicular Height, and the Product is the solid Content.



Let ABC be a Cone, the Diameter of whose Base AB is 26.5 Inches, and the Height of the Cone DC is 16.5 Feet: First, square the Diameter 26.5, and it is 702.25; which multiply by .7854, and the Product is 551.54715; which multiply'd by 5.5, and the Product is 3033.47825; which divided by 144, the Quotient is 21.07, fere, the solid Content of the Cone.

26.5 the Diameter.

26.5

1325

1590

530

702.25 the Square.

.7854

280900

351125

561800

491575

551.541715 Area of the Base.

55 a third Part of the Height.

275773

275773

1443033.503(21.07 Feet, the Content.

153

947

By Scale and Compasses.

Extend the Compasses from 13.54 to 26.5, (the Diameter) that Extent (turn'd twice over from 5.5, a third Part of the Height) will at last fall upon 21.07 Feet, the Content.

To find the superficial Content,

Multiply half the Circumference 41.626 by the slant Height AC 198.46, and the Product is 8261.09596; which divided by 144, the Quotient is 57.37, fere, the Curve-Surface; to which add the Base, the Sum is 61.2 Feet, the superficial Content.

41.626

41.626 the half Circumference of the Base.

198.46 the slant Height.

249756

166504

333008

374634

41626

144)8261.09596(57.37 Feet, *fers.*

3.83 the Base add.

1061

530

989

61.20 the whole Content.

144)551.54(3.83

1195

434

2

By Scale and Compasse.

Extend the Compasses from 144 to 198.46, that Extent will reach from 41.626 to 57.37 Feet, the Curve-Surface.

And extend the Compasses from 12 to 26.5 (the Diameter) that Extent, turn'd twice over from .7854, will at last fall upon 3.83 Feet, the Base; which added to 57.37, the Sum is 61.2 Feet, the superficial Content.

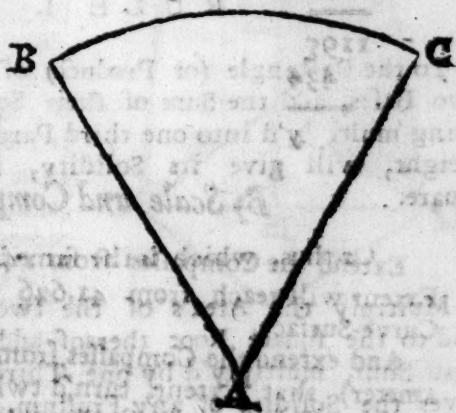
Demonstration. Every Cone is the third Part of a Cylinder of equal Base and Altitude. The Truth of this may easily be conceiv'd, by only considering, that a Cone is but a round Pyramid; and therefore, it must needs have the same Ratio to its circumscribing Cylinder, as the square Pyramid hath to its circumscribing

scribing Parallelopipedon, viz. as 1 to 3. However, to make it yet clearer, let it be farther consider'd, That

Every right Cone is constituted of an infinite Series of Circles, whose Diameters do continually increase in arithmetical Progression, beginning at the Vertex, or Point C, the Area of its Base AB being the greatest Term, and its perpendicular Height DC the Number of all the Terms; therefore the Area of the Circle of the Base, multiply'd by a third Part of the Altitude DC, will be the Sum of all the Series, equal to the Solidity of the Cone, by Lemma III.

The Curve-Superficies of every right Cone, is equal to half the Rectangle of the Circumference of its Base into the Length of its Side.

For the Curve-Surface of every right Cone, is equal to the Sector of a Circle, whose Arch BC is equal to the Periphery of the Base of the Cone, and Radius AB equal to the slant Side of the Cone. Which will appear very evident, if you



cut a Piece of Paper in the Form of a Sector of a Circle, as ABC, and bend both the Sides AB and AC together, 'till they meet, and you will find it to form a right Cone.

I have omitted the Demonstrations touching the Superficies of all the foregoing Solids, because I thought it needless, they being all compos'd of Squares, Parallelograms, Triangles, &c. which Figures are all demonstrated before. And if the Area of all such Fi-

gures as composes the Solid, be found severally, and added together, the Sum will be the superficial Content of the Solid.



§ VII. Of the Frustum of a PYRAMID.

A Frustum of a Pyramid, is the remaining Part, when the Top is cut off by a Plain parallel to the Base. To find the solid Content thereof, there are several Rules.

R U L E I.

To the Rectangle (or Product) of the Sides of the two Bases, add the Sum of their Squares; that Sum being multiply'd into one third Part of the Frustum's Height, will give its Solidity, if the Bases be square.

Or thus, which is the same in Effect.

Multiply the Area's of the two Bases together, and to the square Root thereof add the two Area's; that Sum, multiply'd by one Third of the Height, gives the Solidity of any Frustum, square or mult-angled.

R U L E II.

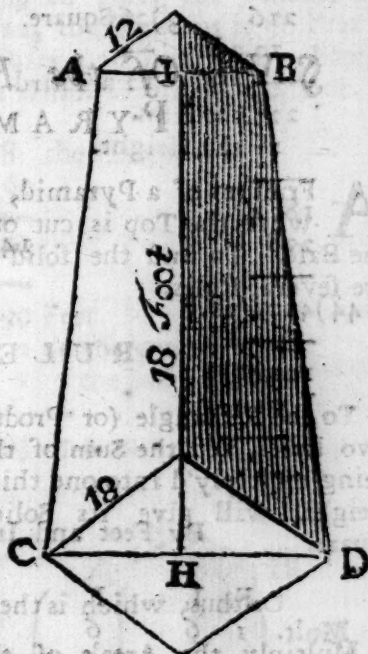
To the Rectangle of the Sides of the two Bases, add one third Part of the Square of their Difference, that Sum being multiply'd into the Height, will produce the Solidity, if the Bases be Squares: But if they be triangular or multangular, the said Rectangle

of

of the Sides, with the third Part of the Square of their Difference, will be the Square of a mean Side; and the Square Root thereof will be such a mean Side as will reduce the tapering Solid to a Prism equal thereunto.

Example. Let ABCD be the Frustum of a square Pyramid, the Side of the greater Base 18 Inches, and the Side of the lesser 12 Inches, and the Height 18 Foot; the Solidity thereof is requir'd.

First, multiply the two Sides together 18 by 12, and the Product is 216, and the Difference of the Sides is 6, whose Square is 36; a third Part thereof is 12, which added to 216, the Sum is 228 Inches, the Area of a mean Base; which multiply'd by 18 Feet, the Length, the Product is 4104; this divided by 144, the Quotient is 28.5 Feet, the solid Content.



Or, by the first Rule, thus; the Square of 18 is 324, and the Square of 12 is 144, and the Rectangle of 18 by 12 is 216; the Sum of these three is 684, which multiply'd by 6, the Product is 4104; which divided by 144, the Quotient is 28.5 Feet, the same as before.

See the Work of both Ways.

18	6 Diff.	18	12
12	6	18	12
216	3)36 Square.	324 Sq.	144 Sq.
12 add	12 a Third.	144	
228 the Sum.		216	
18 the Height.		684 the Sum.	
1824		6 a 3d of the Height.	
228		144)4104(28.5 Feet.	
144)4104(28.5		1224	
1224		720	
720		...	
...		...	

By Feet and Inches thus:

Mult.	F.	I.	I.	
by	1	6	6	
Prod.	1	6	3)36	q.
add	0	1	12	
Mult.	1	7		
By	18	0	Height.	
	18	0		
	9	0		
	1	6		
Content	28	6		

Or thus.

1	3	Sq. of the greater.
1	6	the Rectangle.
1	0	Sq. of the less.
4	9	Triple of a mean Area.
6	0	a 3d of the Height.
28	6	Content.

To find the superficial Content.

The Perimeter of the greater Base is 72, and the Perimeter of the lesser Base is 48; add both the Perimeters together, the Sum is 120; the Half thereof is 60; which multiply'd by 18 Feet, the Product is 1080; this divided by 12, the Quotient is 90 Feet; to which add the two Bases 2.25 Feet, and 1 Foot, the Sum is 93.25 Feet, the whole superficial Content.

18	12	18	the Height.
4	4	60	
72	48	12) 1080	
48			
2) 120			
60			

90 Feet.

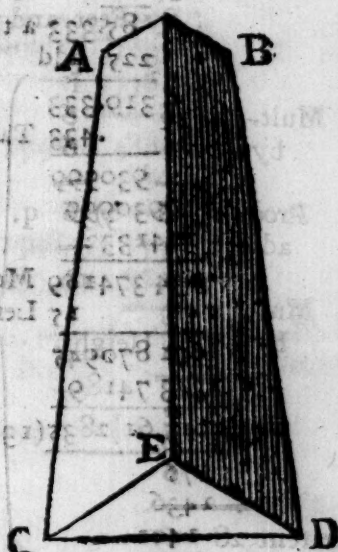
2 25 the greater Base.

1 the lesser Base.

93.25 Sum.

Again, Let ABCD be the Frustum of a triangular Pyramid, each Side of the greater Base 25 Inches, and each Side of the lesser Base 9 Inches, and the Length 15 Feet; the solid Content thereof is requir'd.

By the second Rule, multiply 25 by 9, and the Product is 225; and the Difference between 25 and 9 is 16; which squar'd, makes 256, a third Part thereof is 85.333, which added to 225, and the Sum is 310.333; this multiply'd by .433, the



Product is 134.374, &c. which is the Area of a mean Base; and that multiply'd by 15 Feet, the Length, the Product is 2015.610; which divided by 144, the Quotient is 13.99 Feet, the Solidity.

Or thus, by the latter Part of the first Rule: Find the Area of the greater Base, which you will find to be 270.625, and the Area of the lesser Base will be 35.073; these two Area's multiply'd together, the Product is 9491.630625, the Square Root thereof is 97.425; to which add the two Area's, and the Sum is 403.123; which multiply'd by a third Part of the Length 5, and the Product is 2015.615; and that divided by 144, and the Quotient is 13.99 Feet, as before. See the Working of both.

$$\begin{array}{r}
 25 \quad 25 \\
 9 \quad 9 \\
 \hline
 16 \text{ Diff.} \\
 \text{Product } 225 \quad 16 \\
 \hline
 96 \\
 16
 \end{array}$$

3)256 the Square.

85.333 a third Part.

225. add

310.333

.433 Tabular Number.

930999

930999

1841332

134.374189 Mean Area.

15 Length.

671870945

134374189

144)2015.612835(13.99 Feet.

575

1436

1401

105

Product is 433, which is the Area of the
 Base; and this multiply by the Length
 the Product is 1875, which is divided by
 the Quotient 81 Square. 433
 Or thus: the Area of the Base is 433
 the Area of the Solid is 1875, which will
 be divided by the Area of the Base, and the
 Quotient is 4.33, which is the Length of the
 Solid. 1875
 433
 1875
 1875
 2500
 270.625 Area. 1811875
 1894375
 1353125
 811875
 9491.630625 (97.425
 81

	187) 1391
	1309
	8263
	7776
	48706
	38964
	974225
	974225

Multiplication of the Side by the Length
 Number, and the Result is the Area of the Polygon.

For finding the Content.

the Length of the greater Side is 75, and the
 Length of the lesser Side is 27; the Sum of both is
 102, and the half Sum is 51; which multiply by 75
 3825

270.625 greater Area.

97.425 the mean Proportional.

35.073 the lesser Area.

403.123 the Triple of a mean Area.

5 a third Part of the Height.

144) 2015.615 (13.99 Feet, the Solidity.

575

1436

1401

105

In finding the Area of the triangular Base, I multiply'd by .433, because that is the Area of the equilateral Triangle, when the Side thereof is 1. A Table of the Area's, or Multipliers, for finding the Area's of Polygons, follows.

Number of Sides.	Names.	Multipliers.
3	Trigon	.433013
4	Tetragon	1.000000
5	Pentagon	1.720477
6	Hexagon	2.598076
7	Heptagon	3.633959
8	Octagon	4.828427
9	Enneagon	6.181824
10	Decagon	7.694209

Multiply the Square of the Side by the tabular Number, and the Product is the Area of the Polygon.

To find the Superficial Content.

The Perimeter of the greater Base is 75, and the Perimeter of the lesser Base is 27; the Sum of both is 102 and the half Sum is 51; which multiply'd by

15 Feet, the Product is 765; which divided by 12, the Quotient is 63.75; to which add the Sum of the two Bases 2.12 Feet, and the Sum is 65.87 Feet, the whole superficial Content.

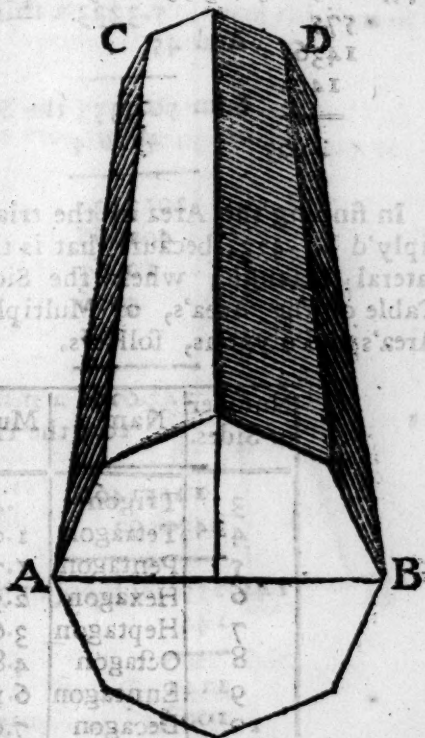
Note, That 51 should have been multiply'd by the slant Height, but the Difference it would make, is but .06 of a Foot, which is inconsiderable.

Again, Suppose ABCD to be the Frustum of a Pyramid, having an octagonal Base, each Side thereof being 9 Inches, and each Side of the lesser Base 5 Inches, and the Length or Height 10.5 Feet, the Solidity is requir'd.

By the second Rule, multiply the greater Side 9, by the lesser Side 5, and the Product is 45; then the Difference between 9 and 5 is 4, which squar'd, makes 16; a third Part thereof is 5.3333, which added to 45, the Sum is 50.3333; multiply this last by the Number in the Table 4.8284, and the Product is 243.0292, the Area of a mean Base; which multiply'd by the Height 10.5 Feet, and the Product is 2551.8066; then divide this last Product by 144, and the Quotient is 17.72 Feet, the solid Content.

See the Work.

Mult.



Mult. 9 Inches. 9 from the greater Side.
by 5 Inches. 5 subtract the lesser.

Prod. 45

4
4
—

316 square

5.3333 a third Part.

Add 45

Sum 50.3333 the Square of a mean Side.

4828.4

2013332

402666

10067

4026

201

243.0292 a mean Area.

10.5 the Height.

12151460

2430292

144)2551.80(17.72

144

1111

1008

8301

8001

300

238

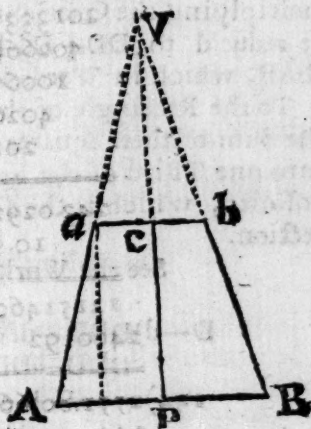
12

To find the superficial Content.

The Perimeter of the greater Base is 72, and the Perimeter of the lesser Base is 40, and their Sum is 112; the half thereof is 56, which multiply'd by the Height 10.5 Feet, and the Product is 588; which divided by 12, the Quotient is 49 Feet; to which add the Sum of the two Bases, 3.55 Feet, and the Sum is 52.55 Feet, the whole superficial Content.

Demonstration. From the Rules deliver'd in the IVth and VIth Sections, the two foregoing Rules may easily be demonstrated.

Suppose a square Pyramid, as ABV, to be cut by a Plain at ab, parallel to its Base AB, and it were requir'd to find the Solidity of the Frustum, or Part a b AB. Let there be given



$D=BA$, the Side of the greater Base.

$d=ba$, the Side of the lesser Base.

$H=CP$, the perpendicular Height.

First	1	$D-d : H :: d : \frac{dH}{D-d} = VC$ by the Figure
then	2	$DD \times \frac{H+VC}{3} =$ the whole Pyramid BVA
and	3	by Section the IVth. $dd \times \frac{1}{3} VC =$ the Pyramid aVb cut off.

Then,

Then, in the 2d and 3d Steps, if instead of VC, you take $\frac{dH}{D-d}$ equal to it by the first Step, it will be,

$$\begin{array}{lcl} \text{viz. 1. 2.} & 4 & \frac{DDD}{3D-3d} = \text{the whole Pyramid BVA.} \\ \text{and 1. 3.} & 5 & \frac{ddd}{3D-3d} = \text{the Pyramid aVb.} \\ & 6 & \frac{DDD-ddd}{3D-3d} = \text{the Frustrum abAB.} \end{array}$$

And by dividing $DDD-ddd$ by $D-d$, and then multiplying the Quotient by $\frac{1}{3}H$, the last Step will be reduc'd to $DD+Dd+dd : \times \frac{1}{3}H =$ the Frustrum abAB, which in Words is thus :

To the Rectangle of the Sides of the two Bases, add the Sum of their Squares ; that Sum being multiply'd into one Third of the Frustrum's Height, will give its Solidity, which is the same as the first Rule of this Section.

See the Work of the Division.

$$\begin{array}{r} D-d)DDD-ddd(DD+Dd+dd \\ \underline{DDD-DDd} \end{array}$$

$$\begin{array}{r} Ddd-ddd \\ \underline{Ddd-Ddd} \end{array}$$

$$\begin{array}{r} Ddd-ddd \\ \underline{Ddd-ddd} \end{array}$$

The same Reason will hold good for all Frustums of Pyramids or Cones, whether the Base be triangular or multangular; because the Squares of the Sides of any Figure, or the Squares of the Diameters of Circles, are proportional to the Area, which proves the latter Part of the said first Rule.

Again, to prove the second Rule,

Suppose $x = D - d$. And $F =$ the Frustum.

then $DD + Dd + dd = \frac{3F}{H}$ by the last.

$1 \text{ } \odot \text{ } 2 \text{ } 3 \text{ } xx = DD - 2 Dd + dd.$

$2 \text{ } + \text{ } 3 \text{ } 4 \text{ } 3 Dd = \frac{3F}{H} - xx.$

$4 \text{ } \div \text{ } 3 \text{ } 5 \text{ } Dd = \frac{F}{H} - \frac{1}{3} xx. \text{ Or } Dd + \frac{1}{3} xx = \frac{F}{H}.$

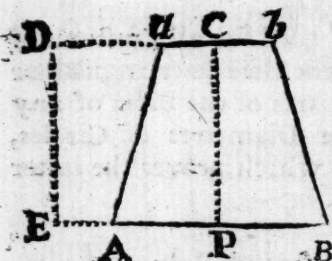
$5 \text{ } \times \text{ } H \text{ } 6 \text{ } Dd + \frac{1}{3} xx \times H = F, \text{ the Frustum } abAB.$

Which in Words is thus :

To the Rectangle of the Sides of the two Bases, add one third Part of the Square of the Difference of the said Sides, and multiply the Sum by the Height of the Frustum, the Product is the Solidity of the Frustum.

The superficial Content of Frustums, (all but the Bases) is compos'd of Trapeziums, so many as the Frustum has Sides; as the Square Frustum $abAB$ in the last Figure is compos'd of four Trapeziums, having the two upper, and also the two lower Angles equal; if therefore the Trapeziums $abAB$ be cut in two by the Line CP , and the two Pieces laid together, the Line bB upon the Line aA , the narrow End of the one, to the broad End of the other, it will form a right-angled Parallelogram, as is plain by the Figure annex'd;

Q



nex'd; the Parallelogram DCEP being equal to the Trapezium abAB; because the Side Da is equal to PB, and EA is equal to aC. Therefore, to find the Area of the Trapezium, add half the Side ab to half the Side AB, and it makes DC or EP; which multiply by the

Height PC, the Product is the Area of the Parallelogram DCEP, equal to the Trapezium abAB; then, if that be multiply'd by the Number of Trapeziums, the Product will be the superficial Content of the Frustum, wanting the Bases. Or if the whole Perimeter of the greater Base be added to the Perimeter of the lesser Base, and half the Sum multiply'd by the Height, the Product will be the superficial Content of the Trapeziums at once.

Note, That half the Sum of the Perimeters should be multiply'd by the slant Height up the Middle of one of the Trapeziums; but in the foregoing Examples, I have multiply'd by the perpendicular Height, because the Difference is very inconsiderable.



§ VIII. Of the Frustum of a CONE.

A Frustum of a Cone, is that Part which remains, when the Top-End is cut off by a plain Parallel to the Base. To find the solid Content, the Rules are the same in Effect as for the Frustum of a Pyramid.

R U L E I.

To the Rectangle of the Diameters of the two Bases, add the Squares of the said Diameters, and multiply the

the Sum by .7854, the Product will be the Triple of a mean Area; which multiply'd by $\frac{1}{3}$ of the perpendicular Height, that Product will be the solid Content.

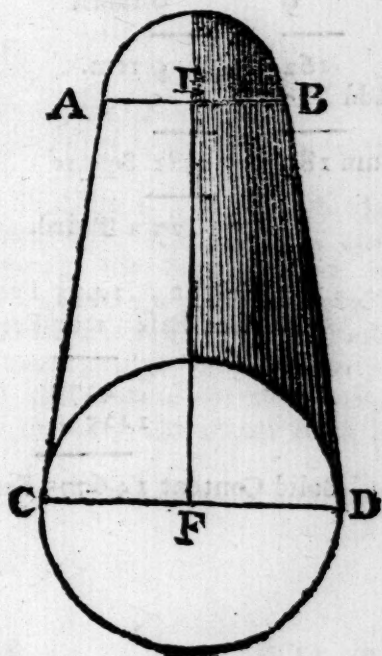
Or thus: Multiply the Area's of the greater and lesser Bases together, and out of the Product extract the square Root, and add the two Area's and square Root together, and multiply the Sum by one Third of the perpendicular Height, the Product is the solid Content.

R U L E II.

To the Rectangle of the greater and lesser Diameters, add one third Part of the Square of their Difference, and multiply the Sum by .7854, the Product is a mean Area; which multiply'd by the perpendicular Height, the Product is the Solidity.

Example. Let ABCD be the Frustum of a Cone, whose greater Diameter CD is 18 Inches, and the lesser Diameter AB 9 Inches, and the Length 14.25 Feet, the solid Content is requir'd.

Multiply 18 by 9, and the Product is 162, and the Difference between 18 and 9, is 9, whose Square is 81, a third Part is 27; which add to 162, the Sum is 189; this multiply'd by .7854, the Product is 148.44; which divided by 144, the Quotient is 1.03



Q 2

Feet,

Feet, the Area of a mean Base; which multiply'd by 14.25 Feet, the Height, the Product is 14.6775 Feet, the solid Content.

Or thus, by the first Rule.

The Square of 18 (the greater Diameter) is 324, and the Square of 9 (the lesser Diameter) is 81, and the Rectangle, or Product of 18 by 9, is 162; the Sum of these three is 567, which multiply'd by .7854, the Product is 445.3218; which divided by 144, the Quotient is 3.09 Feet, the triple Area of a mean Base; this multiply'd by 4.75 Feet, (a third Part of the Height) and the Product is 14.6775 Feet, the Solidity, the same as before.

See the Work.

18	18 from	.7854
9	9 subtr.	189
<u>162</u>	9 rem.	70686
Add 27	9	62832
<u>Sum 189</u>	3)81 Square	7854
	27 a Third.	144)148.44 06(1.03.
		144

Height 14.25 Feet.

Area Base 1.03 Feet.

4275

1425

444

432

12

Solid Content 14.6775 Feet.

324 the Square of 18.

162 the Rectangle.

81 the Square of 9.

567 the triple Square of a mean Diameter.

.7854

567

54978

47124

39270

144)445.32|18 (3.09

475

1332

1545

36

2163

1236

The Solidity 14.6775

To find the superficial Content.

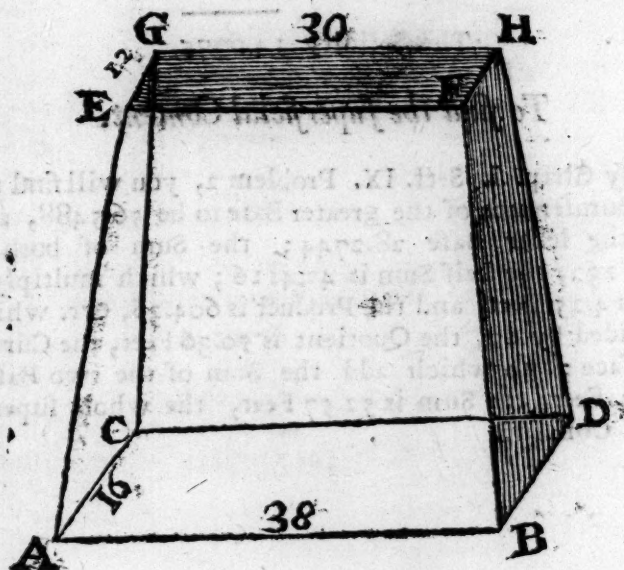
By Chap. I. Sect. IX. Problem 2, you will find the Circumference of the greater Base to be 56.5488, and of the lesser Base 28.2744; the Sum of both is 84.8232, the half Sum is 42.4116; which multiply'd by 14.25 Feet, and the Product is 604.36, &c. which divided by 12, the Quotient is 50.36 Feet, the Curve-Surface; to which add the Sum of the two Bases, 2.21 Feet, the Sum is 52.57 Feet, the whole superficial Content.

§ IX. To measure the Frustum of a rectangled Pyramid, call'd a PRISMOID, whose Bases are parallel one to another, but disproportionall.

The RULE.

TO the greatest Length add half the lesser Length, and multiply the Sum by the Breadth of the greater Base, and reserve the Product.

Then, to the lesser Length add half the greater Length, and multiply the Sum by the Breadth of the lesser Base; and add this Product to the other Product reserv'd, and multiply that Sum by a third Part of the Height, and the Product is the solid Content.



Examples

Example. Let ABCDEFGH be a Prismoid given, the Length of the greater Base AB 38 Inches, and its Breadth AC 16 Inches; and the Length of the lesser Base EF is 30 Inches, and its Breadth 12 Inches, and the Height 6 Feet; the solid Content is requir'd.

To the greater Length AB 38, add half EF the lesser Length 15, the Sum is 53; which multiply'd by 16, the greater Breadth, and the Product is 848; which reserve:

Again, To EF 30, add half AB 19, and the Sum is 49; which multiply by 12, (the lesser Breadth EG) the Product is 588; to which add 848, (the reserv'd Product) and the Sum is 1436; which multiply'd by 2, (a third Part of the Height) and the Product is 2872; divide this Product by 144, and the Quotient is 19.94 Feet, the solid Content.

$$\begin{array}{r} 38 = AB \\ 15 = \frac{1}{2} EF \\ \hline 53 \\ 16 = AC \\ \hline \end{array}$$

$$\begin{array}{r} 318 \\ 53 \\ \hline 848 \\ 588 \\ \hline \end{array}$$

$$1436$$

2 = a third Part of the Heights.

144)2872(19.94 Feet, the Content.

$$\begin{array}{r} 1432 \\ 1360 \\ 640 \\ \hline 64 \end{array}$$

$$\begin{array}{r} 30 = EF \\ 19 = \frac{1}{2} AB \\ \hline 49 \\ 12 = EG \\ \hline \end{array}$$

$$588$$

To prove this Rule, Let us suppose the Solid cut into Pieces, so as to make it capable of being measur'd by the foregoing Rules, thus; Let ABCD represent the greater Base, and EFGH the lesser Base, and let the Solid be suppos'd to be cut thro' by the Lines ac, bd, and ef, gh, from the Top to the Bottom, so will



there be a Parallelopipedon, having its Bases equal to the lesser Base EFGH, and its Height 6 Feet, equal to the Height of the Solid. Multiply 30 (the Length of the Base) by 12, (the

Breadth thereof) and the Product is 360; which multiply'd by the Height 6 Feet, and the Product is 2160. Then there are two Wedge-like Pieces, whose Bases are abEF, and GHcd; if these two Pieces be laid together, the thick End of one to the thin End of the other, they will compose a rectangled Parallelopipedon; which to measure, multiply the Length of the Base 30 by its Breadth 2, and the Product is 60; which multiply'd by 6, (the Height) the Product is 360. Then there are two other Wedge-like Pieces, whose Bases are eEgG, and FfHh; these two laid together, will compose a rectangled Parallelopipedon; to measure this, multiply the Length of the Base 12 by the Breadth 4, the Product is 48; which multiply'd by 6, (the Height) the Product is 288. And lastly, there are four rectangled Pyramids, at each Corner one; which to measure, multiply the Length of one of the Bases 4, by its Breadth 2, the Product is 8; which multiply'd by 2, (a third Part of the Height) the Product is 16; and that multiply'd by 4, (because there are 4 of them) and the Product is 64. Then add all these together, and the Sum is 2872, and divide by 144, the Quotient is 19.94 Feet, the same as before, which shews the Rule to be true.

See the Work.

12	30	12	4
30	2	4	2
360	60	48	8
6	6	6	2
2160	360	288	16
360			4
288			
64			64

144)2872(19.94 Feet, the whole Content.

1432
 1360
 640

 64

To find the superficial Content.

Half the Perimeter of the greater Base is 54, and Half the Perimeter of the lesser Base is 42; which added together, the Sum is 96; which multiply'd by 6, (the Height) the Product is 576: Divide this Product by 12, the Quotient is 48 Feet; to which add the Sum of the two Bases 6.72 Feet, and the Sum is 54.72 Feet, the whole superficial Content.

S X.

§ X. To measure a CYLINDROID; that is, a Frustum of a Cone having its Bases parallel to each other, but unlike.

The R U L E.

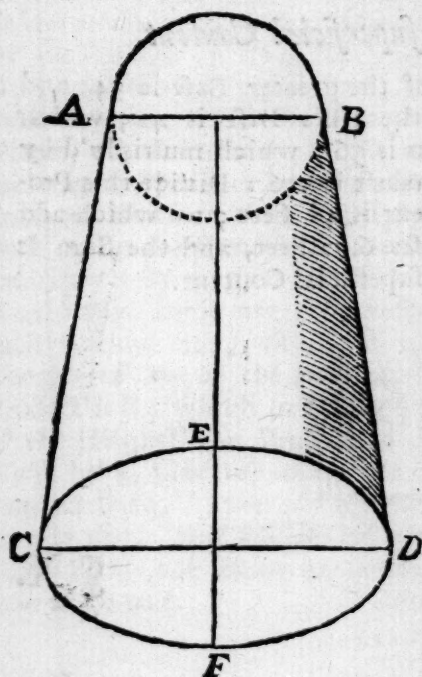
TO the longest Diameter of the greater Base, add half the longest Diameter of the lesser Base, and multiply the Sum by the shortest Diameter of the greater Base, and reserve the Product.

Then, to the longest Diameter of the lesser Base, add half the longest Diameter of the greater Base, and multiply the Sum by the shortest Diameter of the lesser Base, and add the Product to the former reserv'd Sum, and that Sum will be the triple Square of a mean Diameter; which multiply'd by .7854, and

that Product multiply'd by a third Part of the Height, the Product is the solid Content.

Example. Let ABCD be a Cylindroid, whose bottom Base is an Oval, the transverse Diameter being 44 Inches, and the conjugate Diameter 14 Inches; and the upper Base is a Circle, whose Diameter is 26 Inches, and the Height of the Frustum is 9 Feet; the Solidity is requir'd.

To 44 (the greater Diameter of the lower Base) add 13, (half the Diameter



Diameter of the lesser Base) the Sum is 57; which multiply'd by 14, (the conjugate Diameter of the greater Base) the Product is 798; which reserve. Then to 26 (the Diameter of the lesser Base) add 22, (half the transverse Diameter of the greater Base) and the Sum is 48; which multiply'd by 26, (the Diameter of the lesser Base) the Product is 1248; to which add the former reserv'd Product, the Sum is 2046; which multiply'd by .7854, and the Product is 1606.9284; which multiply'd by 3, (a third Part of the Height) the Product is 4820.7852; which divided by 144, the Quotient is 33.47 Feet, the solid Content.

See the Work.

44=CD	26=AB
13=half AB	22=half CD
—	—
57 Sum	48 Sum.
14=EF	26=AB
—	—
228	288
57	96
—	—
798. Product reserv'd.	1248
	798 Add.
	—
	2046
	.7854
	—
	8184
	10230
	16368
	14322
	—
	1606.9284
	3
	—
144)4820.7852(33.47	
	500
	—
	687
	—
	1118
	—
	110

This

This Rule being the same as that in the last Section, the Proof of that may serve as a sufficient Proof of this, if what has been before written, be well consider'd.

To find the superficial Content.

To the Periphery of the Ellipsis 97.41 add the Periphery of the Circle 81.68, and the Sum is 179.09; the half thereof 89.545, multiply'd by 9, the Product is 805.905; which divided by 12, the Quotient is 67.16 Feet, the Curve-Surface: Then the Area of the Ellipsis is 3.36 Feet, and the Area of the Circle is 3.68 Feet; both which added to the Curve-Surface, the Sum is 74.2 Feet, the whole superficial Content.

§ XL. Of a SPHERE or GLOBE.

A Sphere, or Globe, is a round solid Body, every Part of whose Surface is equally distant from a Point within it, call'd its Center; and it may be conceiv'd to be form'd by the Revolution of Semi-circles round its Diameter. To find its Solidity, this is

The R U L E.

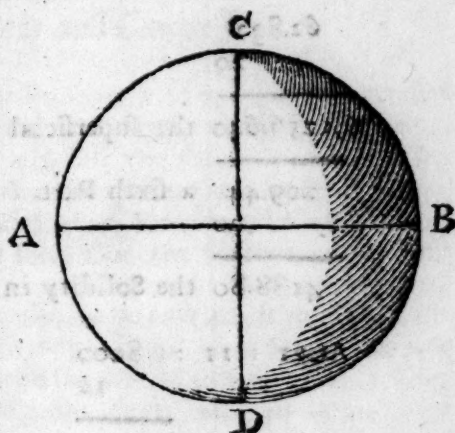
1. Multiply the Axis, or Diameter, into the Circumference, the Product is the superficial Content; which multiply by a sixth Part of the Axis, the Product is the Solidity.

2. Or thus: As 21 : to 11 :: so is the Cube of the Axis to the solid Content.

3. Or, As 1 : is to .5236 :: so is the Cube of the Axis to the solid Content.

Example.

Example. Let ABCD be a Globe, whose Axis is 20 Inches, then the Circumf. will be 62.832: Then, by the first Rule, multiply the Circumference by the Axis, and the Product will be 1256.64, which is the superficial Content in Inches;



es; take a sixth Part thereof, which is 209.44, (because an exact sixth Part of 20 cannot be taken) multiply that sixth Part by 20, (the Axis) and the Product is 4188.8, the Solidity in Inches. Or, if you multiply the superficial Content by the Axis, and take a sixth Part of the Product, the Answer will be the same.

Or thus, by the second Rule:

The Cube of the Axis is 8000, which multiply'd by 11, the Product is 88000; which divided by 21, the Quotient is 4190.47, the Solidity.

Or, by the third Rule:

If the Cube of the Axis be multiply'd by .5236, the Product is 4188.8, the Solidity, the same as by the first Way. If you divide 4188.8 by 1728, the Quotient is 2.424 Feet.

See the Work.

By Scale and Compass 62.832

20

Extend the Compasses from 1 to 20 (the Axis) the Extent (in) will be 62.832 the superficial Content.

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4188.80 the Solidity in Inches.

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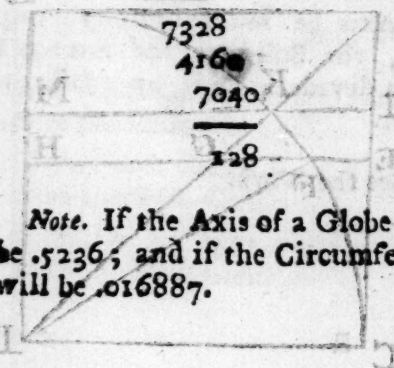
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Note. If the Axis of a Globe be 1, the Solidity will be .5236; and if the Circumference be 1, the Solidity will be .0168887.

By Scale and Compasses.

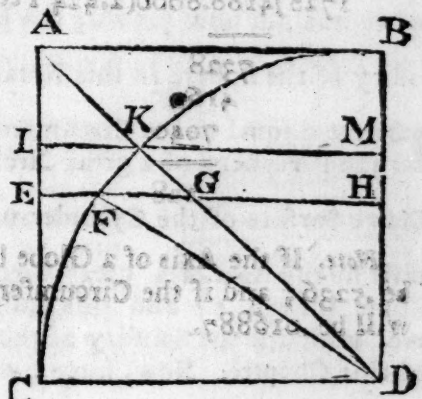
Extend the Compasses from 1 to 20, (the Axis) that Extent (turn'd three times over, from 5236) will at the last fall upon 4188.8, the solid Content in Inches: Or, Extend the Compasses from 1728 to 8000, (the Cube of the Axis) that Extent will reach from 5236 to 2.424, the solid Content in Feet.

Extend the Compasses from 1 to 20, (the Axis) that Extent (turn'd twice over, from 3.1416) will at last fall upon 1256.64, the superficial Content in Inches: Or, Extend the Compasses from 144 to 400, (the Square of the Axis) that Extent will reach from 3.1416 to 8.72, the superficial Content in Feet.

Demonstration. Every Sphere is equal to a Cone whose perpendicular Axis is the Radius of the Sphere, and its Base a Plain equal to all the Surface of it.

For you may conceive the Sphere to consist of an infinite Number of Cones, whose Bases, taken all together, compose the Surface, and whose Vertices meet all together in the Center of the Sphere: Hence the Solidity of the Sphere will be gain'd, by multiplying its Surface by $\frac{1}{3}$ of its Radius.

Let the Square ABCD, the Quadrant CBD, and the right-angled Triangle ABC, be suppos'd all three to revolve round the Line BD as an Axis: Then will the Square generate a Cylinder, the Quadrant an Hemi-



R 2

spheres

sphere, and the Triangle a Cone, all of the same Base and Altitude.

Then the Square of EH ($\equiv \square FD$) $\equiv \square FH + \square DH$ (but $DH \equiv GH$.) And since Circles are as the Squares of their Diameters, (by *Euclid* 12. 2.) the Circle made by the Revolution of EH, must be equal to both the Circles made by the Motions of FH and GH.

If you take the Circle made by the Revolution of FH from both, there will remain the Circle made by the Motion of GH, equal to the Ring describ'd by the Motion of EF. And thus it will always be; wherever you draw the Line EH or IM, &c.

Therefore the Aggregate, or Sum, of all the Rings made by the Revolution of the EF's, must be equal to that of all the Circles made by the Motion of the GH's, i. e. the Dish-like Solid form'd by the revolving Rings, will be equal to the Cone, form'd by the Revolution of the GH's, which are the Elements of the Triangle ABD; that is, the Dish-like Solid will be as the Cone, $\frac{1}{3}$ of the circumscribing Cylinder, and consequently the Hemisphere must be $\frac{2}{3}$ of $\therefore$ Wherefore the Sphere is $\frac{2}{3}$ of the circumscribing Cylinder.

Let the Radius of the Sphere be $r = CD$, then the Diameter will be $2r$; let the Surface of the Sphere generated by the revolving Semicircle, be call'd S , and that of the Cylinder, form'd by the Revolution of $2AC = 2r = \text{Diameter}$, be call'd f . Wherefore, in what was just now prov'd, the Expression for the So-

lidity of the Sphere in this Notation, will be $\frac{rS}{3}$, and

putting c equal to the Circumference of the Base, or for the Periphery of a great Circle of the Sphere, the

Curve-Surface of the Cylinder will be $2rc$; also $\frac{rc}{2}$

will be the Area of a great Circle, (by Sect. IX of Chap. I. Prob. 1.) and this multiply'd by $2r$, makes rrc , which is the Solidity of the Cylinder, by Sect. V of this Chapter. Now, since f was put equal to $2rc =$

to the Curve-Surface of the Cylinder, $\frac{Sr}{2}$ (by substituting f for $2rc$) will be also $=$ to the Solidity of the Cylinder. Now, since the Sphere is $= \frac{2}{3}$ of the Cylinder, $\frac{rS}{3} = \frac{2fr}{3^2}$; that is $\frac{rS}{3} = \frac{2fr}{6} = \frac{fr}{3}$.

Wherefore $rS = rf$, that is, dividing by r , $S = f$; or the Surface of the Sphere is equal to the Curve-Surface of the Cylinder; but the Curve-Surface of the Cylinder was $2rc$.

Wherefore, to find the Area of the Surface of either Sphere or Cylinder, you must multiply the Diameter ($=2r$) by the Circumference of a great Circle of the Sphere, or by the Periphery of the Base. From this

Notation also $\frac{rc}{2}$, the Area of a great Circle of the Sphere, is plainly $\frac{1}{4}$ of $2rc$, the Surface of the Sphere; that is, the Surface of the Sphere is Quadruple of the Area of the greatest Circle of it.

Wherefore, to $2rc$, the convex Surface of the Cylinder, add rc equal to the Area of both its Bases, you will have $3rc$; which shews you, that the Surface of the Cylinder (including its Bases) is to the Surface of the Sphere as 3 to 2; or, that the Sphere is $\frac{2}{3}$ of the circumscribing Cylinder, in Area as well as Solidity.

Or you may prove the Sphere to be $\frac{2}{3}$ of the Cylinder of the same Base and Altitude, by Lemma VI, afore-going, thus;

Let AGB represent the Hemisphere, and AIKB half the Cylinder; then, if the Semi-diameter GH be divided into six equal Parts, and Lines drawn parallel to AB, the Diameter, the Squares of the Semichords, ab , cd , ef , &c.



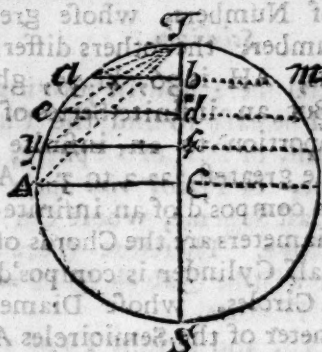
will be a Series of Numbers, whose greatest Term AH is a square Number, the others differing by odd Numbers; that is, AH is 36, kl 37, gh 32, ef 27, ed 20, ab 11: But an infinite Series of such Numbers, are in Proportion to an infinite Number of Terms equal to the greatest, as 2 to 3. And because the Hemisphere is compos'd of an infinite Number of Circles, whose Diameters are the Chords of the Semicircle; and the half Cylinder is compos'd of an infinite Number of Circles, whose Diameters are all equal to the Diameter of the Semicircles AB: Therefore the Hemisphere is in Proportion to the half Cylinder, as 2 to 3; and, consequently, the whole Sphere bears the same Proportion to the whole Cylinder.

That the Superficies of every Sphere (or Globe) is equal to four times the Area of its greatest Circle, is thus prov'd:

The Solidity of the Sphere is constituted of an infinite Number of parallel Circles (as is aforesaid;) consequently, the Superficies of the Sphere will be compos'd of the Peripheries of those Circles which constitute its Solidity.

Note, In the following Demonstrations, $\odot$ signifies any Circle in general; and if any two Letters be join'd to it, thus, $\odot AB$, &c. then it denotes the Area of such a Circle as those two Letters represent the Radius of.

Let



Let $D = TS$, the Axis of any Sphere; then, according to the Property of a Circle, it

will be	1	$D - Tb \times Tb = \square ab;$
that is,	2	$D \times Tb - \square Tb = \square ab;$
therefore	3	$D \times Tb = \square aT.$
For	4	$\square ab + \square Tb = \square aT, (Euc. I. 47.)$
and	5	$D \times Td = \square eT.$
	6	$D \times Tf = \square yT.$

Hence it is evident, that the Series $\square aT, \square cT, \square yT, \&c.$ are in the same Ratio with $Tb, Td, Tf, \&c.$ viz. in arithmetical Progression: Whence it follows, that the $\odot AT =$ to the Sum of all the Circle's Peripheries between T and b .

And $\odot eT =$ the Sum of all the Circle's Peripheries between T and $d, \&c.$

Consequently, that the $\odot AT =$ the Sum of all the Circle's Peripheries included between T and C ; that is, $\odot AT =$ the Superficies of the Semisphere.

And because $\square AC + \square TC = \square AT$, and $\square AC$ is equal to $\square TC$; therefore $\odot AT = 2\odot AC$, is the Superficies of the Semisphere.

Consequently, $4\odot AC$ will be the Superficies of the whole Sphere. Which was, $\&c.$

Scholium.

Scholium.

From the Method here us'd in proving the whole Superficies, it will be easy to find the Curve-Superficies of any Frustum, or Part of a Sphere, that is cut off by a right Line, or Plain, viz. such as the Frustum aTm in the last Scheme, whose Curve-Superficies is $\odot aT$, as above. Therefore (because $\square ab + \square Tb = \square aT$) it will be $\odot ab + \odot Tb =$ the Curve-Superficies of that Frustum.

But if the Axis TS , and Height Tb of the Frustum, are given, then it will be $TS \times Tb = \square aT$, as in the third Step above, which gives the Proportion or Theorem following, viz.

As the Axis of the Sphere : is to the whole Superficies of the Sphere :: so is the Height of any Frustum : to its Curve-Superficies.

To which if there be added the Area of the Frustum's Base, the Sum will be the whole Superficies of the Frustum.

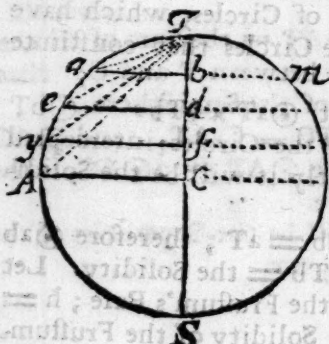
That the Solidity of every Sphere is two Thirds of its circumscribing Cylinder, may be thus prov'd.

According to the Work above, it appears, that $\odot ab$, $\odot ed$, $\odot yf$, &c. do constitute the Solidity of the Sphere; and that $\square aT$, $\square eT$, $\square yT$, &c. are a

Series of Terms in Arithmetical Progression, $\square AT$ being the greatest Term, and TC the Number of Terms; therefore $\odot AT \times \frac{1}{2} TC =$ the Sum of all the Series, by Lemma 2.

And because $\square aT = \square Tb + \square ab$, $\square eT = \square Td + \square ed$, $\square yT = \square Tf + \square yf$, $\square AT = \square TC + \square AC$, &c.

wherein



wherein $\square Tb$, $\square Td$, $\square Tf$, &c. are a Series of Squares, whose Roots Tb , Td , Tf , are in arithmetical Progression; $\square TC$ being the greatest Term, and TC the Number of Terms; therefore $\odot TC \times \frac{1}{3} TC =$ the Sum of all that Series, by Lemma 3.

Consequently, $\odot AT \times \frac{1}{3} TC = \odot TC \times \frac{1}{3} TC =$ the Sum of all the Series $\odot ab$, $\odot ed$, $\odot yf$, &c. which constitute the Solidity of the half Sphere ATG . Put $D = \frac{1}{2} TC$ the Axis of the Sphere; then $\frac{1}{2} D = \frac{1}{4} TC$, and $\frac{1}{3} D = \frac{1}{6} TC$. And because $\square AT = 2 \square TC$, therefore $\odot AT = 2 \odot TC = 1.5704 DD$; & $1.5708 DD \times \frac{1}{4} D = 0.3927 DDD$.

Again, $\odot TC \times \frac{1}{3} TC = 0.7854 DD \times \frac{1}{2} D = 0.1309 DDD$, then $0.3927 DDD - 0.1309 DDD = 0.2618 DDD$, the Solidity of the half Sphere.

Consequently, $0.2618 DDD \times 2 = .5236 DDD$ will be the solid Content of the whole Sphere, which is equal to $\frac{2}{3}$ of the Cylinder; the Diameter of whose Base, and also its Height, is $= D$.

For $0.7854 DDD =$ the Solidity of the Cylinder by Sect. V. But $\frac{2}{3}$ of $0.7854 DDD = 0.5236 DDD$, as before.

Scholium.

From this Demonstration it will be easy to deduce or raise Theorems for finding the solid Content of any Frustum of a Sphere, as aTm in the last Figure.

For we there suppose the Frustum aTm to be constituted of an infinite Series of Circles, which have the same Ratio with all those Circles that constitute the half Sphere.

Therefore it follows, that $\odot aT \times \frac{1}{3} Tb = \odot bT \times \frac{1}{3} Tb$ will be the Sum of all the Circles intercepted between T and b ; consequently it will be the Solidity of that Frustum.

And because $\square ab + \square Tb = aT$; therefore $\odot ab + \odot Tb \times \frac{1}{3} Tb = \odot Tb \times \frac{1}{3} Tb =$ the Solidity. Let $c = ab$ half the Diameter of the Frustum's Base; $h = Tb$ its Height; and $S =$ the Solidity of the Frustum. Then $\odot ab = 3.1416 cc$, and $\odot Tb = 3.1416 hh$; consequently,

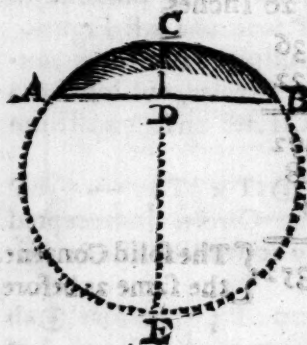
$$\text{quently, } \frac{3.1416cc}{2} + \frac{3.1416hhh}{8} - \frac{3.1416hhh}{3} = S.$$

Which being reduc'd, will become $3cch + hhh \times 0.5236 = S$; which is one Theorem for finding the Solidity of the Frustum, and may be express'd in Words thus:

If to three times the Square of the Semidiameter of the Frustum's Base, you add the Square of the Height of the Frustum, and multiply the Sum by the Height of the Frustum, that Product multiply'd by .5236, the Product will be the solid Content.

But if the Axis of the Sphere, and the Height of the Frustum, be given; then put D = the Axis, h = the Height of the Frustum, and c as before; it will be $D - h = c$, viz. $Dh - hh = cc$. Then will $3Dhh - 2hhh = 3cch + hhh$; consequently $3Dhh - 2hhh \times 0.5236 = S$, the Frustum's Solidity. Which is another Theorem for finding the Solidity of the Frustum, and may be express'd in Words thus:

From three times the Axis, subtract twice the Height of the Frustum, and multiply the Remainder by the Square of the Height, and that Product multiply by .5236, this last Product will be the Solidity of the Frustum.



Example. Let ABCD be the Frustum of a Sphere; suppose AB, the Diameter of the Frustum's Base, be 16 Inches, and CD the Height 4 Inches; the Solidity is requir'd.

By

By the first Rule.

8
8
64 Square of the Semidiameter AD.
3
16 Add the Square of CD.
208
4 multiply'd by CD.
32
5236
822
10472
15708
41888
The Solidity 435.6352

By the second Rule thus.

First, By the Rule in Page 113, you will find the Axis of the whole Globe to be 20 Inches.

20 Axis.
5236
822
10472
15708
41888
Rem. 52
Mult. 16 Sq. of CD.
435.6352

The Solid Content,
the same as before.

312
52
Prod. 832

§ XII. Of a SPHEROID.

A Spheroid is a Solid resembling an Egg. To find the solid Content thereof, this is

The RULE.

Multiply the Square of the Diameter of the greatest Circle, by the Length, and that Product multiply again by .5236, this last Product will be the Solidity of the Spheroid.



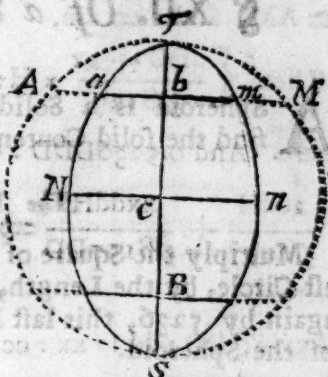
Let AB be the Diameter of the greatest Circle 33 Inches, and CD (the Length) 55 Inches; the Solidity is requir'd.

33	59895
33	5236
—	—
99	359370
99	179685
—	—
1089	119790
55	299475
—	—
5445	31361.0220 the Solidity.
5445	
—	
59895	

Demonstration.

Demonstration. Every Spheroid is equal to $\frac{2}{3}$ of a Cylinder, whose Base is equal to the greatest Circle of the Spheroid, and its Height equal to the Length of the Spheroid.

Suppose the Figure NTnSN in the annex'd Scheme, to represent a Spheroid, form'd by the Rotation of the Semi-Ellipsis TNS, about its transverse Axis TS.



Let $D = TS$, the Length of the Spheroid, and the Axis of its circumscribing Sphere; and $d = Nn$ the Diameter of the greatest Circle of the Spheroid.

Then, because $\square TC : \square NC :: \square Ab : \square ab$, by Sect. XV, Step. 3, Page 123.

Therefore it will be $DD : dd :: \square Ab : \square ab$.

But the Sum of an infinite Series of such Circles as $\odot Ab$ (whose Diameters are Chords) do constitute the Solidity of the Sphere. (By Sect. XI.)

And the Sum of an infinite Series of such Circles as $\odot ab$ (viz. whose Diameters are Ordinates of the Ellipsis) do constitute the Solidity of the Spheroid.

Therefore $DD : dd :: 0.5236DDD : 0.5236Ddd =$ the Solidity of the Spheroid. (By Euc. 5. 12.)

But $0.5236Ddd = \frac{2}{3}$ of the Cylinder, whose Diameter is $= d$, and Height $= D$. (By Sect. V.)

Now, from this Proportion between the Sphere and its inscrib'd Spheroid, it will be very easy to deduce Theorems for finding the solid Content, either of the Frustum or middle Zone of any Spheroid, having the same Height with that of the Sphere; for,

As the Solidity of the whole Sphere : is to the Solidity of the whole Spheroid :: so is any Part of the Sphere : to the like Part of the Spheroid. (By Euc. 5. 12.)

As for Instance : Suppose it was requir'd to find the middle Zone of any Spheroid.

Let $D = TS$, and $d = Nn$, as above ; and $H = bB$, $x = AM$, and $c = am$.

Then $\left\{ \frac{2DD + xx}{3.8127} \times H = \text{the Middle Zone of the} \right.$

Sphere. And $0.5236DDD : 0.5236ddd :: \frac{2DD + xx}{3.8127} \times$

$H : \frac{2ddH}{3.8197} + \frac{xxddH}{3.8197DD} = \text{the middle Zone of the}$
Spheroid.

Again, $DD : dd :: xx : cc$. Therefore $\frac{xxdd}{DD} = cc$.

Consequently, $\frac{xxdd}{DD} \times \frac{H}{3.8127} = \frac{cc}{3.8127} \times H$. Which

being taken instead of $\frac{xxddH}{3.8197DD}$, there will arise

this following Theorem, $\left\{ \frac{2dd + cc}{3.8197} : x H = \text{the} \right.$
middle Zone of the Spheroid.

Note, That $3.8197 = 1.2732 \times 3$. See Page 102.

§ XIII. Of a Parabolick CONOID.

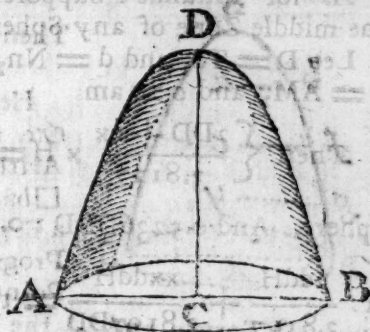
A Parabolick Conoid is something like an half Spheroid, having its Sides somewhat fraiter. It is generated by supposing a Semi-Parabola turn'd about its Axis. To find the solid Content thereof, this is

The R U L E.

Multiply the Square of the Diameter of its Base by .7854, and multiply that Product by half the Height, that last Product shall be the solid Content.

Let

Let ABCD be a Parabolick Conoid, the Diameter of whose Base is 36 Inches, and its Height CD 33 Inches; the Solidity is requir'd.



36	.7854	1017.8784
36	1296	33
216	47124	30536352
108	70686	30536352
1296	15708	2) 33589.9872
	7854	
	1017.8784	16794.9936

1728) 16794.99 | 36 (9.71 Feet, the Content.

15552

12429

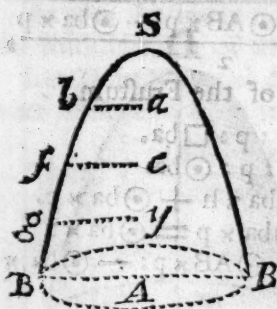
12096

3339

1728

1611

Demonstration. The Parabolick Conoid is constituted of an infinite Number of Circles, whose Diameters are the Ordinates of the Parabola. Now, according to the Property of every Parabola, it will be $SA : AB :: AB : \frac{\square AB}{SA} = L$, the *Latus Rectum*.



$$\text{Then } \begin{cases} Sa \times L = \square ba, \\ Se \times L = \square fe, \\ Sy \times L = \square gy, \text{ \&c.} \end{cases}$$

Here $Sa \times L$, $Se \times L$, $Sy \times L$, &c. are a Series of Terms in Arithmet. Progresf. Therefore $\square ba$, $\square fe$, $\square gy$, &c. are also a Series of Terms in the same Progression, beginning at the Point S, wherein $\square AB$ is the greatest Term, and SA the Number of all the Terms. Therefore $\square AB \times \frac{1}{2} SA =$ the Sum of all the Series. (By Lemma 2.)

Consequently, $\odot AB \times \frac{1}{2} SA =$ the Sum of all the Series of $\odot ba$, $\odot fe$, $\odot gy$, &c. which do constitute the Solidity of the Conoid.

Put $D = 2AB$, and $H = SA$.

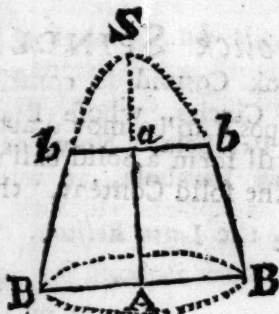
Then $.7854DD \times \frac{1}{2} H = .3927DDH$ will be the solid Content of the Conoid; which is just half the Cylinder, whose Base is $= D$, and Height $= H$.

This being rightly understood, it will be easy to raise a Theorem for finding the lower Frustum of any Parabolick Conoid.

For supposing $h = aA$, the Height of the Frustum, and $p = Sa$, the Height of the Part LSb cut off. Then $h + p = SA$, the Height of the whole Conoid.

Consequently $\frac{\odot AB \times h + \odot AB \times p}{2} =$ the Solidity of the whole Conoid.

And $\frac{\odot ba \times p}{2} =$ the Solidity of the Part cut off.



Therefore

		$\{ \frac{\odot AB \times h + \odot AB \times p - \odot ba \times p}{2},$
Therefore	1	the Solidity of the Fruustum.
But	2	$h + p : \square AB :: p : \square ba.$
Consequ.	3	$h + p : \odot AB :: p : \odot ba.$
3	4	$\odot AB \times p = \odot ba \times h + \odot ba \times p.$
4 — $\odot ba \times p$	5	$\odot AB \times p - \odot ba \times p = \odot ba \times h.$
1 $\times \bar{2}$	6	$\{ \odot AB \times h : + \odot AB \times p : - \odot ba \times p$
		$= 2F.$
6 — 5	7	$\odot AB \times h = 2F : - \odot ba \times h.$
7 + $\odot ba \times h$	8	$\odot AB \times h : + \odot ba \times h = 2F.$
8 $\div \bar{2}$	9	$\{ \frac{\odot AB + \odot ba}{2} : \times h = F, \text{ the Fru-}$
		stum's Solidity.

Let $D = 2AB$, as before, and $d = 2ba$, the Diameter of the Part cut off; then we shall have this following Theorem.

$0.3927 DD + 0.3927 dd : \times h =$ the Solidity of the Fruustum requir'd: Which in Words is thus:

Multiply the Sum of the Squares of the greater and lesser Diameters, by .3927, and that Product by the Height of the Fruustum, the last Product shall be the solid Content.



§ XIV. Of a Parabolick SPINDLE.

IF an acute Parabola be suppos'd to be mov'd about its greatest Ordinate, it will form a Solid call'd a Parabolick Spindle. To find the solid Content, this is

The R U L E.

Multiply the Square of the Diameter of its greatest Circle, by .41888, (being $\frac{1}{3}$ of .7854) and that Product by its Length; that last Product is the solid Content.

Let ABCD be a Parabolick Spindle, whose greatest Diameter CD is 36 Inches, and its Length AB 99 Inches; the Solidity is requir'd.

$$\begin{array}{r} 36 = \text{CD.} \quad .41888 \\ 36 \quad \quad \quad 1296 \end{array}$$

$$\begin{array}{r} 216 \quad \quad \quad 251328 \\ 108 \quad \quad \quad 376992 \\ \hline \quad \quad \quad 83776 \end{array}$$

$$3296 \text{ Square} \quad 41888$$

$$542.86848$$

$$99$$

$$488581632$$

$$488581632$$

$$3728)53743.97952(31.10184$$

$$1903$$

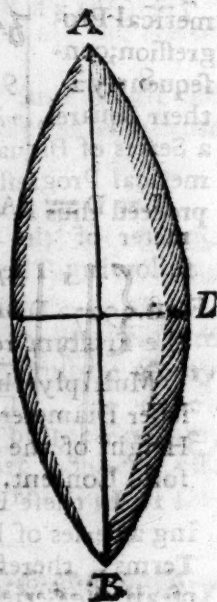
$$1779$$

$$3179$$

$$14515$$

$$6912$$

The solid Content is 31.1084 Feet.



Demonstration.

3. And the $\square fm$, $\square gn$, $\square hp$, &c. will be a Series of Terms in the Ratio of Biquadrats, as above, $\square SA$ being the greatest Term, and AB the Number of all the Terms. Therefore it will be $\frac{\square SA \times AB}{5}$

= the Sum of all the Series, by Lemma V.

Whence it follows, that $\square SA \times AB = \frac{2 \square SA \times AB}{3}$

$+ \frac{\square SA \times AB}{5} =$ the Sum of all the Series of $\square ma$, $\square ne$, $\square py$, &c.

That is, $\frac{8 \square SA \times AB}{15} =$ the Sum of all the Series,

$\square ma$, $\square ne$, $\square py$, &c. Consequently, $\frac{8 \odot SA \times AB}{15}$

= the Sum of all the Series of Circles, $\odot ma$, $\odot ne$, $\odot py$, &c. which constitute the Solidity of half the Spindle, viz. of SAB .

Therefore putting $D = 2SA$, and $H = 2AB$, it will be $0.41888DDH =$ the Solidity of the whole Parabolick Spindle LSB , being $\frac{8}{15}$ of $0.7854DDH$, the Solidity of its circumscribing Cylinder.

From hence we may also raise a Theorem for finding the Frustum, $SAPy$, of the last Figure.

For $\odot SA$ being the greatest Term, $\odot py$ the least Term, and Ay the Number of all the Terms or Circles included between A and y ;

Therefore

Therefore { $\square SA - \frac{2SA \times hp}{3} + \frac{\square hp}{5} : Ay = z$
 the Sum of all the Series, $\square SA$, $\square ma$,
 $\square gn$, $\square py$.

$$1 \times 3 \quad 2 \quad 3 \quad \square SA - 2SA \times hp + \frac{3 \square hp}{5} : xAy = 3z.$$

$$2 \div Ay \quad 3 \quad 3 \quad \square SA - 2SA \times hp + \frac{3 \square hp}{5} = \frac{3z}{Ay}$$

But $4 \quad \square SA - 2SA \times hp = \square py - \square hp$, per Step. 6.

$$3-4 \quad 5 \quad 2 \square SA + \frac{3 \square hp}{5} = \frac{3z}{Ay} - \square py + \square hp.$$

$$5 + \&c. \quad 6 \quad 2 \square SA + \square py - \frac{2}{3} \square hp = \frac{3z}{Ay}.$$

Consequ. $7 \quad 2 \odot SA + \odot py - \frac{2}{3} \odot hp : x \frac{1}{3} Ay = z$,
 the Sum of all the Series of $\odot SA$, $\odot ma$, $\odot ne$, $\odot py$,
 which do constitute the Solidity of the Frustum $SApy$.
 Therefore, putting $D = 2SA$, as before, $C = 2py$,
 $x = 2hp$, and $H = Ay$, it will be $1.5708DD + .7854CC$
 $- .31416xx : x \frac{1}{3} H =$ the Frustum $SApy$. And if
 we make $L = 2H$, then $1.5708DD + .7854CC -$
 $.31416xx : x \frac{1}{3} L =$ the Double of that Frustum, be-
 ing the middle Zone. Which in Words is thus:

Multiply the Square of the greatest Diameter by
 1.5708 , and multiply the Square of the lesser Diame-
 ter by $.7854$, and multiply the Square of the Differ-
 ence of the Diameters by $.31416$; from the Sum of
 the two former Products subtract the latter Product,
 and multiply the Remainder by one third Part of the
 Length, and that Product will be the Solidity of the
 middle Zone requir'd.

Pen, and the remaining Figure are the Quotient
and the Figures cut off are Feet. I put 100 divided



CHAP. III.

*Of the measuring of the Works of
the several Artificers relating to
Building; and what Methods
and Customs are observ'd therein.*

§ I. Of CARPENTERS Work.

THE Carpenters Works, which are measurable,
are Flooring, Partitioning, and Roofing; all
which are measur'd by the Square of 10 Feet long,
and 10 Feet broad; so that one Square contains 100
square Feet.

1. Of Flooring.

If a Floor be 57 Feet 3 Inches long, and 28 Feet 6
Inches broad; how many Squares of Flooring is there
in that Room?

Multiply 57 Feet 3 Inches by 28 Feet 6 Inches, and
the Product is 1631 Feet, &c. which divide by 100;
this is done by cutting off from the Product two Fi-
gures towards the right Hand with a Dash of the
Pen,

Chap. III. CARPENTERS Work. 203

Pen, and the remaining Figures are the Quotient, and the Figures cut off are Feet : Thus, 1631 divided by 100, by cutting off 31 from the right Hand thereof, the Quotient is 16 Squares, and the 31 cut off is 31 Feet.

See the Work, both by Decimals, and also by Feet and Inches.

57.25	F.	I.
28.5	57	3
	28	6
28625		
45800	456	
11450	114	
	28	7 6
	7	0 0
16 31.625	16 31	7 6

Facit 16 Squares and 31 Feet.

Note, That .5 is the Decimal for a half of any Thing, .25 is the Decimal for a quarter, and .125 is the Decimal for half a quarter; so, in the last Example, .25 is the Decimal of 3 Inches, because 3 Inches is a quarter of a Foot; and .5 is the Decimal of 6 Inches, because 6 Inches is half a Foot.

Example 2. Let a Floor be 53 Feet 6 Inches long, and 47 Feet 9 Inches broad; how many Squares are contain'd in that Floor?

47.75	F.	I.
53.5	53	6
	47	9
23875		
14325	371	
23875	212	
	26	9
	13	4 6
	23	6
25 54.625	25 54	7 6

Facit 25 Squares and 54 Feet.

By

By Scale and Compasses.

In the first Example, extend the Compasses from 1 to 28.5, that Extent will reach from 57.25 to 16 Squares and near a third Part.

In the second Example, extend the Compasses from 1 to 47.75, that Extent will reach from 53.5 to 25 Squares and above a half.

2. Of Partitioning.

Example 1. If a Partition between Rooms be in Length 82 Feet 6 Inches, and in Height 12 Feet 3 Inches, how many Squares are contain'd therein?

The Length and Breadth being multiply'd together, the Product is 1010.625; which divide by 100, (as before is shew'd) and the Answer is 10 Squares 10 Feet; the Inches or Parts in these Cases, is of no Value.

12.25	F. I.
82.5	82 6
6125	12 3
2450	990 0
9800	20 7 6
1010.625	1010 7 6

Facit 10 Squares 10 Feet.

Example 2. If a Partition between Rooms be in Length 91 Feet 9 Inches, and in Breadth 11 Feet 3 Inches, how many Squares are contain'd therein?

The Length and Breadth being multiply'd together, the Product is 1032 Feet; which divided by 100, the Answer will be 10 Squares and 32 Feet.

91.75	F. 1.
11.25	91 9
	11 3
45875	
18350	1009 3
9175	22 11 3
9175	
	10 32 2 3
10 32.1875	

3. Of Roofing.

It is a Rule amongst Work-men, that the Flat of any House, and half the Flat thereof taken within the Walls, is equal to the Measure of the Roof of the same House; but this is when the Roof is true pitch'd. For if the Roof be more flat or steep than the true Pitch, it will measure to more or less accordingly.

Example 1. If a House, within the Walls, be 44 Feet 6 Inches long, and 18 Feet 3 Inches broad; how many Squares of Roofing will cover that House?

Multiply the Length and Breadth together, and the Product is 812 Feet, the Flat; the half thereof is 406 Feet, which added to the Flat, the Sum is 1218 Feet; which divided by 100, the Answer is 12 Squares and 18 Feet.

T 18.25

18.25	F.	I.
44.5	44	6
<u> </u>	18	3
91.25	<u> </u>	
7360	352	
<u>7360</u>	44	
	11	1 6
Flat 812.425	<u> </u>	
half 406	9	0 0
<u> </u>		
12 18	The Flat 812	1 6
	The half 406	

Sum 12|18

Facit 12 Squares 18 Feet.

By Scale and Compasses.

In the first Example of Partitioning, extend the Compasses from 1 to 12.25, that Extent will reach from 82.5 to 10 Squares and one Tenth.

In the second Example, extend the Compasses from 1 to 11.25, that Extent will reach from 91.75 to 10 Squares, and a little less than a third Part.

In the Example of Roofing, extend the Compasses from 1 to 48.25, that Extent will reach from 44.5 to 812 the Flat; to which add the half thereof, and the Sum is 12.18, which is 12 Squares 18 Feet, as above.

There are other Works about a Building, done by the Carpenter, that are measur'd by the Foot, running Measure, that is, by the Number of Feet in Length only; as Cornices, Doors and Cases, Window-Frames, Guttering, Lintels, Sommers, Skirt-Boards, &c.

Note 1. In the measuring of Flooring, after you have measur'd the whole Floor, you must deduct out of it the Well-Holes for the Stairs and Chimneys; and in Partitioning, for the Doors, Windows, &c. except (by Agreement) they are to be included.

Note

Note 2. In measuring of Roofing, seldom any Deductions are made for the Holes for the Chimney-Shafts, the Vacancies for Lutheren-lights and Sky-lights; for they are more Trouble to the Work-man, than the Stuff which would cover them is worth.

§ II. Of BRICK-LAYERS Work.

THE principal is Tiling, Walling, and Chimney-Work.

I. Of Tiling.

Tiling is measur'd by the Square of 10 Feet, as Flooring, Partitioning, and Roofing were in the Carpenter's Work; so that between the Roofing and Tiling, the Difference will not be much, yet the Tiling will be the most; for the Brick-layers sometimes will require to have double Measure for Hyps and Vallies. When Gutters are allow'd double Measure, the Way is to measure the Length along the Ridge-Tile, and by that Means the Measure of the Gutters becomes double; it is usual also to allow double Measure at the Eaves, so much as the Projector is over the Plate, which is commonly about 18 or 20 Inches.

Example 1. There is a Roof cover'd with Tiles, whose Depth on both Sides (with the usual Allowance at the Eaves) is 37 Feet 3 Inches, and the Length 45 Feet; I demand how many Squares of Tiling are contain'd therein?

F.	I.	
37	3	37.25
45	0	45
		18625
185		14900
148		
11	3	1676.25
1676	3	

Answer, 16 Squares 76 Feet.

Example 2. There is a Roof cover'd with Tiles, whose Depth, on both Sides, (with the Allowance at the Eaves) is 35 Feet 9 Inches, and the Length 43 Feet 6 Inches; I demand how many Squares of Tiling are in the Roof?

F.	I.	
43	6	35.75
35	9	43.5
		17875
215		10725
129		14300
21	9	
10	10	6
17	6	
1555	1	6

Here the Length and Depth being multiply'd together, the Product is 1555 Feet; which divided by 100, (as before is taught) the Answer is 15 Squares and 55 Feet.

By Scale and Compasses.

In the first Example extend the Compasses from 1 to 37.25, that Extent will reach from 45 to 16 Squares and a little above three quarters of a Square.

In

In the second Example, extend the Compasses from 1 to 35.75, that Extent will reach from 43.5 to 15 Squares and 55 Feet, that a little above a half Square.

2. Of Walling.

Brick-layers commonly measure their Work by the Rod-square of 16 Feet and a half; so that one Rod in Length, and one in Breadth, contains 272.25 square Feet; for 16.5, multiply'd in it self, produces 272.25 square Feet. But in some Places the Custom is to allow 18 Feet to the Rod; that is, 324 square Feet. And in some Places the usual Way is, to measure by the Rod of 21 Feet long and 3 Feet high; that is, 63 square Feet; and here they never regard the Thickness of the Wall, but the usual Way is to moderate the Price according to the Thickness.

When you measure a Piece of Brick-work, the first Thing is to enquire by which of those Ways it must be measur'd; then, having multiply'd the Length and Breadth in Feet together, divide the Product by the proper Divisor, either for Rods or Roods, and the Quotient is square Rods or square Roods accordingly.

But commonly Brick-walls, that are measur'd by the Rod, are to be reduc'd to a standard Thickness, viz. of a Brick and a half thick, (if it be not agreed on to the contrary); and to reduce a Wall to Standard-Thickness, this is

The RULE.

Multiply the Number of superficial Feet that are found to be contain'd in any Wall, by the Number of Half-Bricks which that Wall is in Thickness; one third Part of that Product shall be the Content thereof in Feet, reduc'd to the Standard-Thickness of one Brick and a half.

Example 1. If a Wall be 72 Feet 6 Inches long, and 19 Feet 3 Inches high, and 5 Bricks and a half thick, how many Rods of Brick-work are contain'd therein, when reduc'd to the Standard.

19.25 Height.

72.5 Length.

9625

3850

13475

1395.625

11

3)15351.875

272.25) 5117.291 (18 Rods.

239479

68.06) 21679 (3 quarters of a Rod.

12.61

Answer, 18 Rods, 3 Quarters, 12 Feet.

$$\begin{array}{r}
 \text{F. I.} \\
 72 \ 6 \\
 19 \ 3 \\
 \hline
 648 \\
 72 \ 6 \\
 18 \ 1 \ 8 \\
 \hline
 9 \ 6 \ 0 \\
 \hline
 1395 \ 7 \ 6 \\
 11 \\
 \hline
 3)15345 \\
 \hline
 272)5115(18 \text{ Rods.} \\
 \hline
 2395 \\
 \hline
 68)219(3 \text{ quarters of a Rod.} \\
 \hline
 15
 \end{array}$$

Note, That 68.06 is one fourth Part of 272.25.

Note also, That in reducing of Feet into Rods, they usually reject the odd Parts, and divide only by 272, as is done in the second Way of the last Example; so the Answer, by that second Way, is 18 Rods, 3 Quarters, and 15 Feet, more by about $2 \frac{1}{2}$ Feet than by the first Way, where it is done decimally; a Thing very insignificant.

Example

Example 2. If a Wall be 245 Feet 9 Inches long, and 16 Feet 6 Inches high, and 2 Bricks and a half thick; I demand how many Rods of Brick-work are contain'd therein, when reduc'd to Standard-Thickness?

245.75

16.5

122875

147450

24575

4054.875

5

3)20270

272)6756(24 Rods.

1316

69)128(3 quarters of a Rod.

24

Answer, 24 Rods, 3 Quarters, 14 Feet.

F. I.

245 9

16 6

1470

245

122 10 6

12 0 0

4054 10 6 Answer in Feet.

Before I shew how to work the two last Examples by Scale and Compasses, I will shew how to find proper Divisors to facilitate the Operation, because it would be too intricate and tedious to perform by Scale and Compasses, according to the Rules above taught.

To find proper Divisors.

Divide 3 (the Number of half Bricks in $1\frac{1}{2}$) by the Number of half Bricks in the Thickness, the Quotient will be a Divisor which will give the Answer in Feet. But if you would have a Divisor to bring the Answer in Rods at once, then multiply 272.25 by the Divisor found for Feet, and the Product will be a Divisor which will give the Answer in Rods.

Example. Let it be requir'd to find a Divisor proper to reduce a Wall of 3 Bricks thick.

Divide 3 by 6, (the half Bricks in the Thickness) and the Quotient is .5, which is a Divisor that will give the Answer in Feet. Then multiply 272.25 by 5, and the Product is 136.125, the Divisor which will give the Answer in Rods; that is, as 136.125 is to the Length of the Wall, so is the Height to the Content in Rods. Or, as .5 is to the Length, so is the Height to the Content in Feet.

After the same Manner may you find Divisors for any other Thickness, which you will find to be as express'd in the following little Table.

The Thickness of the Wall.	Divisors for the Answer in Feet.	Divisors for bringing the Answer in Rods.
1 Brick thick	1.5	408.375
1 $\frac{1}{2}$ Brick thick	1.	272.25
2 Bricks thick	.75	204.1875
2 $\frac{1}{2}$ Bricks thick	.6	163.35
3 Bricks thick	.5	136.125
3 $\frac{1}{2}$ Bricks thick	.4285	116.678
4 Bricks thick	.375	102.0937

Let the second Example foregoing be wrought by Scale and Compasses, where the Length is 245.75, the Height 16.5, and the Thickness 2 $\frac{1}{2}$ Bricks.

Extend the Compasses from 163.35 (the tabular Number against 2 $\frac{1}{2}$ Bricks) to 245.75, that Extent will reach from 16.5 to 24 Rods and 8 Tenths,

Again, if the Length be 75 Feet 6 Inches, and the Height 18 Feet 9 Inches, at 3 $\frac{1}{2}$ Bricks thick, how many Rods are contain'd therein?

Extend the Compasses from 116.678 (the tabular Number) to 18.75, that Extent will reach from 75.5 to 12.13; that is, 12 Rods and a little above half a Quarter.

It will be very proper and commodious for such as have frequent Occasion to measure Brick-work, to have in the Line of Numbers little Brass Center-Pins at each of the Numbers in the third Column of the above little Table, with a Figure to denote the Thickness of the Wall.

If

If a Wall be 104 Feet 9 Inches long, and 17 Feet 3 Inches high, how many Roods are contain'd therein?

	F.	I.
104-75	104	9
17-25	17	3
52375	728	
20950	104	
73325	26	2 3
10475	12	9 0
63)1806.9375(28	1806	11 3
126		

Answer, 28 Roods 42 Feet.

546

504

42

Note, That such as dig Cellars, do many times do them by the Floor, 18 Feet square, and a Foot deep, being a Floor of Earth; that is, 324 solid Feet.

3. Of Chimneys.

If you are to measure a Chimney standing alone by it self, without any Party-Wall being adjoin'd, then girt it about for the Length, and the Height of the Story is the Breadth; the Thickness must be the same as the Jaums are of, provided that the Chimney be wrought upright from the Mantle-tree to the Cieling, not deducting any thing for the Vacancy between the Floor (or Hearth) and the Mantle-tree, because of the Gatherings of the Breast and Wings, to make Room for the Hearth in the next Story.

If the Chimney-back be a Party-wall, and the Wall be measur'd by it self, then you must measure the Depth of the two Jaums, and the Length of the Breast for a Length, and the Height of the Story the Breadth, at the same Thickness your Jaums were of.

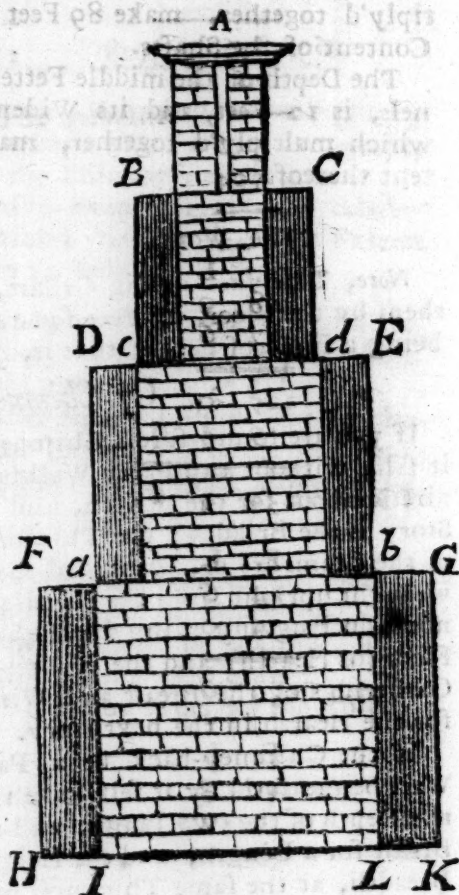
When you measure Chimney-Shafts, girt them with a Line round about the least Place of them, for the

the Length, and the Height shall be your Breadth : And if they be four-Inch Work, then you must set down your Thickness at one Brick-Work ; but if they be wrought 9 Inches Thick, (as sometimes they are, when they stand high and alone above the Roof) then you must account your Thickness $1\frac{1}{2}$ Brick, in Consideration of Wyths, and Pargetting, and Trouble in Scaffolding.

It is customary in most Places to allow a double Measure for Chimneys.

Example. Suppose this Figure ABCDEFGHK, to be a Chimney that hath a double Tunnel towards the Top, and a double Shaft, and is to be measur'd according to double Measure.

First, I begin with the Breast-Wall IL, and the two Angles LK and HI, which together are 18 Feet 9 Inches ; then take the Height of the Square HF, 12 Feet 6 Inches, which multiply'd together, produce 234 Feet 4 Inches 6 Parts, for the Content of the Figure FGHK.



For the Square DaEb, the Length of the Breast-Wall and two Angles, is 14 Feet 6 Inches, and the Height Da 9 Feet; which multiply'd together, make 130 Feet 6 Inches, for the Content of the Part DaEb.

Then the Height of the next Square 7 Feet, and the Length of the Breast-Wall and two Angles is 10 Feet 3 Inches; which multiply'd together, produceth 71 Feet 9 Inches, for the Content of the Square BcCd.

The Compass of the Chimney-Shafts is 13 Feet 9 Inches, and the Height 6 Feet 6 Inches; which multiply'd together, make 89 Feet 4 Inches 6 Parts, the Content of the Shafts.

The Depth of the middle Fetter, that parts the Fun- nels, is 12 Feet, and its Wideness 1 Foot 3 Inches; which multiply'd together, make 15 Feet, the Con- tent thereof.

The Work.

F. I.

18 9

12 6

225 0

9 4 6

FGHK 234 4 6

F. I.

14 6

9 0

DaEb 130 6

F. I.

10 3

7 0

BcCd 71 9

18.75

12 5

9375

3750

1875

FGHK 234.375

14.5

9

DaEb 130.5

10.25

7

BcCd 71.75

F. I.

13	9	13.75
6	6	6.5
82	6	68.75
6	10	6
		8250

The Shaft 89 4 6

The Shaft 89.375

F. I.

1	3	1.25
12	0	12
The Fetter	15 0	The Fetter 15.00

F. I. P.

272	1082 (3 Rods.	FGHK	234	4	6
		DaEb	130	6	0
68)	266 (3 Quarters.	BcCd	71	9	0
		The Shaft	89	4	6
Rem.	62 Feet.	The Fetter	15	0	0

The Sum 541 0 0

The Double 1082 0 0

Having added the five Products together, and doubled the Sum, that double Sum is the Content of the Chimney in Feet, according to double or customary Measure; which Feet must be reduc'd to Rods, as was shew'd before.

So the Feet in the foregoing Example being reduc'd to Rods, (the Thickness being suppos'd $1\frac{1}{2}$ Bricks) it makes 3 Rods 3 Quarters and 62 Feet; that is, 4 Rods wanting 6 Feet.

This is all the Measure that can be allow'd, when the Chimney stands in a Gavel, or Side-Wall; in which Case the Back of the Chimney (here not measur'd) is accounted as Part of the Gavel; but if the Chimneys stand by themselves, as all Stacks of Chimneys

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Chimneys in great Buildings do, which, in such Case, is all Chimney-Work; and therefore ought to be measur'd double on all Sides.

§ III. Of PLAISTERERS Work.

THE Plaisterers Works are principally of two Kinds, namely, 1. Works lath'd or plaister'd, which they call Cieling. 2. Works render'd; which is of two Kinds, viz. upon Brick-Walls, or between Quarters, in the Partitions between Rooms; all which are measur'd by the Yard square, or Square of 3 Feet, which is 9 Feet.

I. Of Cieling.

If a Cieling be 59 Feet 9 Inches long, and 24 Feet 6 Inches broad, how many Yards doth that Cieling contain?

Multiply 59 Feet 9 Inches by 24 Feet 6 Inches, and the Product is 1463 Feet 10 Inches 6 Parts; which divided by 9, the Quotient is 162 Yards 5 Feet.

F.	I.		59.75
59	9		24.5
24	6		—
			29875
236			23900
118			11950
	29	10	6
	18	0	0
			—
			9)1463.875
1463	10	6	

Answer 162 5

By Scale and Compasses.

Extend the Compasses from 9 to 59 Feet 9 Inches, that Extent will reach from 24 Feet 6 Inches, to 162.5 Yards.

2. Of Rendering.

Example. If the Partitions between Rooms be 141 Feet 6 Inches about, and 11 Feet 3 Inches high, how many Yards are in those Partitions?

Multiply 141 Feet 6 Inches by 11 Feet 3 Inches, and the Product is 1591 Feet 10 Inches 6 Parts; which divided by 9, gives 176 Yards 7 Feet, the Answer.

141 6	141.5
11 3	11.25
1556 6	7075
35 4 6	2830
9)1591 10 6	1415
	1415
Answer 176 7	9)1591.875
	176.87

Answer 176.87 Yards.

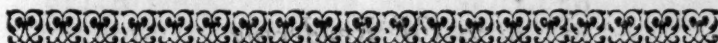
Extend the Compasses from 9 to 141.5, that Extent will reach from 11.25 to 176.87 Yards.

Note 1. If there be any Doors, Windows, or the like, in your Partitioning, you must make Deductions for them.

Note 2. When you measure Rendering upon Brick-Walls, you are to make no Deductions; but when you measure Rendering between Quarters, you may very well deduct one fifth Part, for the Quarters, Braces, and Entertieces.

Note

Note 3. That Whiting and Colouring are both measur'd by the Yard, as Cieling and Rendering were; and as in Rendering between Quarters, you deducted one fifth Part, so in Whiting and Colouring, you must add one fourth, or one fifth Part at least.



§ IV. Of JOYNEERS Work.

JOYNEERS do measure their Work by the Yard Square; but in taking their Dimensions, they differ from some others; for they have a Custom, and say, We ought to measure where our Plane touches; wherefore, in taking the Height of any Room, where there is a Cornish about, and swelling Pannels and Mouldings, they, with a String begin at the Top, and girt-over all the Mouldings; which will make the Room to measure much higher than it is: Then for measuring about the Room, they only take it as it is upon the Floor.

Example 1. If a Room of Wainscot (being girt downwards over the Mouldings) be 15 Feet 9 Inches high, and 126 Feet 3 Inches in Compass; how many Yards doth that Room contain?

Multiply the Compass by the Height, and the Product is 1988 Feet 5 Inches 3 Parts; which divided by 9, gives 220 Yards and 8 Feet, the Answer.

F.	I.	
126	3	126.25
15	9	15.75
630		63125
126		88375
63	16	63125
31	69	12625
3	90	
9)1988	4375	
9)1988	53	
220	8	

Answer 220 8

Facit 220 Yards 8 Feet.

Example 2. If a Room of Wainscot be 16 Feet 3 Inches high, (being girt over the Mouldings) and the Compass of the Room 137 Feet 6 Inches, how many Yards are contain'd therein?

Multiply 137 Feet 6 Inches, by 16 Feet 3 Inches, and the Product is 2234 Feet 4 Inches 6 Parts; which divided by 9, the Quotient is 248 Yards and 2 Feet.

F.	I.	
137	6	137.5
16	3	16.25
830		6875
137		2750
34	46	8250
9)2234	46	1375
248	2	9)2234375
		248 2

Facit 248 Yards 2 Feet.

By

By Scale and Compasses.

For the first Example, extend the Compasses from 9 to 126.25, that Extent will reach from 15.75 to 220.9 Yards.

For the second Example, extend the Compasses from 9 to 137.5, that Extent will reach from 16.25 to 248 Yards, and about a Quarter.

In Joyner's Work there is another Thing to be observ'd, that is, in the measuring of Doors, Window-Shutters, and all such Work as is wrought on both Sides, they are paid for Work and half Work; so that in Measuring all such Work, you must first find the Content as before, and take half that Content, and add to it; so shall the Sum be the Content at Work and half.

Example. If the Window-Shutters about a Room be 69 Feet 9 Inches broad, and 6 Feet 3 Inches high; how many Yards are contain'd therein, at Work and half?

Multiply 69 Feet 9 Inches by 6 Feet 3 Inches, and the Product is 435 Feet 11 Inches 3 Parts; the half whereof is 217 Feet 11 Inches 7 Parts; which added together, the Sum is 653 Feet 10 Inches 10 Parts; which divided by 9, the Quotient is 72 Yards 5 Feet, the Content at Work and half.

F.	I.	
69	9	69.75
6	3	6.25
418	6	34875
17	5 3	13950
435	11 3	41850
217	11 7	
9) 653	10 10	435.9375
72	5	217.9687
		653.9062

Facit 72 Yards 5 Feet.

By

By Scale and Compasses.

Extend the Compasses from 9 to 69.75, that Extent will reach from 6.25 to 48 4 Yards; the Half whereof is 24.2; which added together, make 72.6 Yards, the Content at Work and half.

Note, That you must make Deductions for all Window-Lights; but you must measure the Window-Boards, Sopheta Boards, and Cheeks, by themselves.

§ V. Of PAINTERS Work.

THE taking the Dimensions of Painters Work, is the same as that of Joyners, by girting over the Mouldings and swelling Pannels, in taking the Height; and it is but Reason that they should be paid for that on which their Time and Colour are both expended. The Dimensions thus taken, the casting up, and reducing Feet into Yards, is altogether the same as the Joyners Work; but the Painter never requires Work and half, but reckons his Work once, twice, or thrice colour'd over. Only take Notice, that Window-Lights, Window-Bars, Casements, and such like Things, they do at so much *per Piece*.

Example. If a Room be painted, whose Height (being girt over the Mouldings) is 16 Feet 6 Inches, and the Compass of the Room 97 Feet 9 Inches, how many Yards are in that Room?

Multiply 97 Feet 9 Inches by 16 Feet 6 Inches, and the Product is 1612 Feet 10 Inches 6 Parts; which being divided by 9, the Quotient is 178 Yards and 10 Feet.

F.	I.		97.75
97	9		16.5
16	6		
			48875
584			58650
98			9775
48	10	6	
			1612.875
9	1612	10 6	
179	1		

Facit 179 Yards 1 Foot.

By Scale and Compaffes.

Extend the Compaffes from 9 to 16.5, that Extent will reach from 97.75 to 179.2 Yards.



§ VI. Of GLASIERS Work.

GLasiERS do measure their Work by the Foot Square; so that the Length and Breadth of a Pane of Glas in Feet being multiply'd into each other, produceth the Content.

Note, That GlasiERS do usually take their Dimensi-
ons to a quarter of an Inch; and in multiplying
Feet, Inches, and Parts, the Inch is divided into 12
Parts, as the Foot is, and each Part subdivided into
12, &c.

Example 1. If a Pane of Glas be 4 Feet 8 Inches
and 3 Quarters long, and 1 Foot 4 Inches 1 Quarter
broad, how many Feet of Glas is in that Pane?

The Decimal of $\left\{ \begin{array}{l} 8 \text{ Inches } \frac{3}{4} \\ 4 \text{ Inches } \frac{1}{4} \end{array} \right\}$ is $\left\{ \begin{array}{l} .729 \\ .354 \end{array} \right\}$

F. I.

F. I. P.

4 8 9

1 4 3

4 8 9

1 6 11 0

1 2 2 3

6 4 10 2 3

4.729

1.354

18916

23645

14187

4729

6.403066

Answer, 6 Feet 4 Inches.

By Scale and Compasses.

Extend the Compasses from 1 to 1.354, that Extent will reach from 4.729 to 6.4 Feet, the Content.

Example 2. If there be 8 Panes of Glafs, each 4 Feet 7 Inches 3 Quarters long, and 1 Foot 5 Inches 1 Quarter broad, how many Feet of Glafs is contain'd in the said 8 Panes?

The Decimal of $\left\{ \begin{array}{l} 7 \text{ Inches } \frac{3}{4} \\ 5 \text{ Inches } \frac{1}{4} \end{array} \right\}$ is $\left\{ \begin{array}{l} .646 \\ .437 \end{array} \right\}$

F. I. P.

4 7 9

1 5 3

4 7 9

1 11 2 9

1 1 11 3

6 8 1 8 3

8

53 5 1 6 0

4.646

1.437

32522

13938

18584

4646

6.676302

8

53.410416

Facit 53 Feet 5 Inches.

By Scale and Compasses.

Extend the Compasses from 1 to 1.437, that Extent will reach from 4.646 to 6.676; then extend the Compasses from 1 to 8, that Extent will reach from 6.676 to 5.34, the Content.

Example 3. If there be 16 Panes of Glass, each 4 Feet 5 Inches and a half long, and 1 Foot 4 Inches 3 Quarters broad, how many Feet of Glass is contain'd therein?

F. I. P.	4.458
4 5 6	1.395
1 4 9	
	22290
4 5 6	40122
1 5 10 0	13374
3 4 1 6	4458
6 2 8 1 6	6.218910
4	4
24 10 8 6 0	24.87564
4	4
99 6 10 0 0	99.50256

Facit 99 Feet 6 Inches.

Note, That instead of multiplying by 16, I have multiply'd by 4 twice, because 4 times 4 is 16.

By Scale and Compasses.

Extend the Compasses from 1 to 1.395, that Extent will reach from 4.458 to 6.219; then extend the Compasses from 1 to 16, that Extent will reach from 6.219 to 99.5 Feet, the Content.

Note, That when Windows have half Rounds at the Top, they measure them at the full Height, as if they were square. Also round or oval Windows are measured

fur'd at the full Length and Breadth of their Diameters. Likewise Crocket-Windows in Stone-work are all measur'd by their full Squares. And there is Reason for so doing; for the Trouble in taking their Dimensions to work by, the Waste of Glafs in working, and the Time expended in setting up, is far more than the Glafs can be valu'd at.



§ VII. Of MASONS Work.

Masons do measure their Work sometimes by the Foot solid, sometimes by the Foot superficial; and in some Places they measure their Walling by the Rood; that is, 21 Feet long, and 3 Feet high, which is 63 square Feet. Examples of each are as follows.

Example 1. If a Wall be 97 Feet 5 Inches long, 18 Feet 3 Inches high, and 2 Feet 3 Inches thick, how many solid Feet are contain'd in that Wall?

F.	I.	
97	5	97.417
18	3	18.25
776		487085
97		194834
24	4 3	779336
6	0 0	97417
1	6 0	
		1777.86025
1777	10 3	2.25
2	3	
		888930125
3555	8 6	355572050
444	5 6 9	355572050
4000	2 0 9	4000.1855625

Mul-

Multiply the Length, Height, and Thicknefs together, and the laſt Product is 4000 Feet 2 Inches, the ſolid Feet contain'd in the Wall.

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 18.25, that Extent will reach from 97.417 to 1777.86; then extend from 1 to 1777.86, that Extent will reach from 2.25 to 4000.18, the ſolid Content.

Example 2. If a Wall be 107 Feet 9 Inches long, and 20 Feet 6 Inches high, how many Feet ſuperficial is contain'd therein?

F.	I.	
107	9	107.75
20	6	20.5
		53875
2155	0	21550
53	10 6	
		2208.875
2208	10 6	

Facit 2208 Feet 10 Inches:

By Scale and Compaſſes.

Extend the Compaſſes from 1 to 107.75, that Extent will reach from 20.5 to 2208.875, the ſuperficial Feet.

Example 3. If a Wall be 112 Feet 3 Inches long, and 16 Feet 6 Inches high, how many Roods are contain'd therein?

F.	I.	
112	3	112.25
16	6	16.5
676	0	56125
112		67350
		11225
56	1 6	
1852	1 6	63)1852.125(29
		592

Facit 29 Roods 25 Feet.

25

By Scale and Compasses.

Extend the Compasses from 63 to 16.5, that Extent will reach from 112.25 to 29.4 Roods, the Content.





C H A P. IV.

The Measuring of B O A R D *and*
T I M B E R.



§ I. Of BOARD-MEASURE.

TO measure a Board, is no other but to measure a long Square.

Example 1. If a Board be 16 Inches broad and 13 Feet long, how many Feet is contain'd therein?

Multiply 16 by 13, and the Product is 208; which divided by 12, gives 17 Feet, and 4 remains, which is a third Part of a Foot.

Or thus: Multiply 156 (the Length in Inches) by 16, and the Product is 2496; which divided by 144, the Quotient is 17 Feet, and 48 remains, which is a third Part of 144, the same as before.

As 12 : 13 :: 16

13

48

16

12)208

17 $\frac{4}{12}$

Or, As 144 : 156 :: 16

16

936

156

144)2496(17 $\frac{4}{12}$

1056

48

By Scale and Compasses.

Extend the Compasses from 12 to 13, that Extent will reach from 16 to 17 $\frac{1}{3}$ Feet, the Content.

Or, extend from 144 to 156, (the Length in Inches) that Extent will reach from 16 to 17 $\frac{1}{3}$ Feet, the Content.

Example 2. If a Board be 19 Inches broad, how many Inches in Length will make a Foot?

Divide 144 by 19, and the Quotient is 7.58 very near; and so many Inches in Length, if a Board be 19 Inches broad, will make a Foot.

Inch. Inch, Inch Inch.

As 19 : 144 :: 1 : 7.58 *ferè.*

Extend

Extend the Compasses from 19 to 144, that Extent will reach from 1 to 7.58; that is, 7 Inches and something more than a half. So if a Board be 19 Inches broad, if you take 7 Inches and a little more than a half with your Compasses from a Scale of Inches, and run that Extent along the Board, from End to End, you may find how many Feet that Board contains, or you may cut off from that Board any Number of Feet desir'd.

For this Purpose there is a Line upon most ordinary Joint-Rules, with a little Table plac'd upon the End, of all such Numbers as exceed the Length of the Rule, as in this little Table annex'd.

0	0	0	0	5	0	8 $\frac{1}{2}$	6
12	6	4	3	2	2	1	1
1	2	3	4	5	6	7	8

Here you see, if the Breadth be 1 Inch, the Length must be 12 Feet; if 2 Inches, the Length is 6 Feet; if 5 Inches broad, the Length is 2 Feet 5 Inches, &c.

The rest of the Lengths are express'd in the Line, thus, if the Breadth be 9 Inches, you will find it against 16 Inches, counted from the other End of the Rule; if the Breadth be 11 Inches, then a little above 13 Inches will be the Length of a Foot, &c.



§ II. Of SQUAR'D TIMBER.

BY squar'd Timber is here meant all such as have equal Bases, and the Sides strait and parallel. The Rules for measuring all such Solids, is shew'd in Section II of Chap. II; to which I refer you.

Example 1. If a Piece of Timber be 1 Foot 3 Inches (or 15 Inches) square, and 18 Feet long, how many solid Feet are contain'd therein?

$ \begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ 18 \\ \hline 1800 \\ 225 \\ \hline 144)4050(28.125 \\ \hline 1170 \\ \hline 180 \\ 360 \\ 720 \end{array} $	<table style="border: none;"> <tr><td style="text-align: right;">P.</td><td style="text-align: left;">I.</td></tr> <tr><td style="text-align: right;">1</td><td style="text-align: left;">3</td></tr> <tr><td style="text-align: right;">1</td><td style="text-align: left;">3</td></tr> <tr><td colspan="2"> </td></tr> <tr><td style="text-align: right;">1</td><td style="text-align: left;">3</td></tr> <tr><td style="text-align: right;">3</td><td style="text-align: left;">9</td></tr> <tr><td colspan="2"> </td></tr> <tr><td style="text-align: right;">1</td><td style="text-align: left;">6</td></tr> <tr><td style="text-align: right;">2</td><td style="text-align: left;">6</td></tr> <tr><td colspan="2"> </td></tr> <tr><td style="text-align: right;">9</td><td style="text-align: left;">4</td></tr> <tr><td style="text-align: right;">5</td><td style="text-align: left;">6</td></tr> <tr><td colspan="2"> </td></tr> <tr><td style="text-align: right;">28</td><td style="text-align: left;">1</td></tr> <tr><td style="text-align: right;">6</td><td></td></tr> </table>	P.	I.	1	3	1	3			1	3	3	9			1	6	2	6			9	4	5	6			28	1	6	
P.	I.																														
1	3																														
1	3																														
1	3																														
3	9																														
1	6																														
2	6																														
9	4																														
5	6																														
28	1																														
6																															

Answer, 28 Feet and half a quarter.

Here, instead of multiplying by 18, (where I wrought by Feet and Inches) I multiply'd by 6, and then by 3, because 3 times 6 is 18.

Example 2. If a Piece of squar'd Timber be 2 Feet 9 Inches deep, and 1 Foot 7 Inches broad, and 16 Feet 9 Inches long, how many Feet of Timber are in that Piece?

Multiply the Depth, Breadth, and Length together, and the Product will be the Content.

	F. I.
33	2 9
19	1 7
—	—
297	2 9
33	1 7 3
—	—
627	4 4 3
16.75	16 9
—	—
3135	69 8 0
4389	3 3 2 3
3762	—
627	72 11 2 3
—	—
144)10502.25(72.93	
—	
422	
1342	
465	
—	
33	

Answer, 72 Feet 11 Inches; or, 72 Feet 29 Parts.

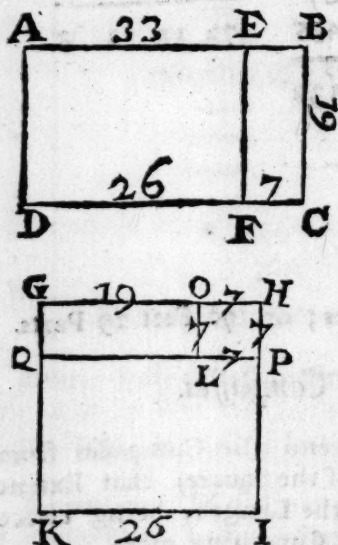
By Scale and Compasses.

For the first Example, extend the Compasses from 12 to 15 Inches, (the Side of the Square) that Extent will reach from 18 Feet, (the Length, being twice turn'd over) to 28 Feet and something more.

For the second Example, find a mean Proportional between 19 Inches and 33 Inches, by dividing the Space between them into two equal Parts; and the Compass-Point will rest upon 25, which is a mean Proportional between 19 and 33.

Then extend the Compasses from 12 to 25, (the Proportional found) that Extent will reach (being twice turn'd over) from 16.75 Feet, the Length, to 72.93 Feet, the Content.

A common Error is committed, for Want of Art, in measuring these last Sorts of Solids, by adding the Depth and Breadth together, and taking half for the Side of a mean Square. This Error, though it be but small, when the Depth and Breadth are pretty near equal; yet if the Difference be great, the Error is very considerable; for the Piece of Timber, thus measur'd, will be more than the Truth by a Piece, whose Length is equal to the Length of the Piece of Timber to be measur'd, and the Square equal to half the Difference of the Breadth and Depth, as I shall here demonstrate.



I say, the Square GHIL is greater than the Parallelogram ABCD, by the little Square OHPL; for the Parallelogram QPIK is equal to the Parallelogram AEFD; and the Parallelogram GOLQ is equal to the Parallelogram EBCF. Therefore the Square is greater than the Parallelogram by the little Square OHPL.

Which was to be prov'd.

Otherwise, you may prove it by Numbers, thus; the Sum of 33 and 19 is 52; the half there-

of is 26; the Square of 26 is 676; and the Product of the Depth and Breadth is 627; the Difference of these two is 49, equal to the Square of half the Difference; for the Difference between 33 and 19 is 14; the half thereof is 7, whose Square is 49. Which was to be prov'd.

Now, if this 49 be multiply'd by the Length of the Piece, and that Product divided by 144, to bring it to Feet, and those Feet added to the true Content, the

Sum

Sum will be equal to the Content found by the false Way mention'd.

See the Work of both.

33 Depth. 16.75 the Length.
19 Breadth. 49 the Square of $\frac{1}{2}$ Diff.

52 Sum. 15075
6700
26 half. 144820.75(5.69
26

156 1007
52 1435
67.6 139
16.75

3380
4732
4056
676
14411323.00(78.63

1243
910
460
28

Feet.

To 72.93 the true Content,
add 5.69 the Part superficial.

Rem. 78.62 equal to the Content by the false Way.

By Feet and Inches.

F. I.	F. I.
0 7	2 2
0 7	2 2
0 4 1	4 4
16 9	4 4
5 5 4	4 8 4
3 0 9	16 9
5 8 4 9	75 1 4
72 11 2 3	3 6 3
Part superfluous.	
true Content add.	
78 7 7 0	Falfe C. 78 7 7
equal to the Content by the falfe Way.	

To find how much in Length makes a Foot of any squar'd Timber.

Always divide 1728 (the solid Inches in a Foot) by the Area of the Base ; the Quotient is the Length of a Foot.

This Rule is general for all Timber, which is of equal Thickness from End to End, whether it be square, triangular, multangular, or round.

Example 1. If a Piece of Timber be 18 Inches square, how much in Length will make a Foot solid?

$$\begin{array}{r}
 18 \\
 18 \\
 \hline
 144 \\
 18 \\
 \hline
 324 \overline{) 1728} (5 \frac{1}{3} \\
 1620 \\
 \hline
 108
 \end{array}$$

Answer, 5 Inches and $\frac{1}{3}$.

By Scale and Compasses.

Extend the Compasses from 1 to 18, that Extent will reach from 18 to 324, the Square or Area of the Base; then extend from 324 to 1728, that Extent will reach down from 1 to 5 Inches and $\frac{1}{3}$ of an Inch.

Or thus. Extend the Compasses from 18 to 41.569, that Extent, turn'd twice over from 1, will at last fall upon $5\frac{1}{3}$, as before.

Note, That 41.569 is the square Root of 1728.

Example 2. If a Piece of Timber be 22-Inches deep and 15 Inches broad, how much in Length will make a Foot?

$$\begin{array}{r}
 22 \\
 15 \\
 \hline
 110 \\
 22 \\
 \hline
 330 \quad 1728(5.23 \\
 \hline
 780 \\
 1200 \\
 \hline
 210
 \end{array}$$

Answer, 5 Inches and .23 Parts.

By Scale and Compasses.

Extend the Compasses from 1 to 15, that Extent will reach from 22 to 330; then extend from 330 to 1728, that Extent will reach from 1 to 5.23 Inches, the Length of a Foot.

There

There is a Line for this Purpose upon most ordinary Rules, with a little Table at the End of all such Numbers as exceed the Length of the Rule, such as this annex'd.

0	0	0	0	9	0	11	3	9	Inches.
144	36	16	9	5	4	2	2	1	Feet.
1	2	3	4	5	6	7	8	9	Side of the Sq.

Here you see, if the Side of the Square be 1, the Length must be 144 Feet; if 2 Inches be the Side of the Square, it must be 36 Feet in Length, to make a solid Foot, &c.

If the Side of the Square be not in the little Table, you will find it upon the Line; thus, if the Side of the Square be 16 Inches, you will find it against 6 Inches and 7 Tenths, counted from the other End of the Rule.

Then, if you take the Length of a Foot from the Line of Inches with your Compasses, and run the Compasses along the Piece, from End to End, you will find how many Feet are contain'd in that Piece; or you may cut off any Number of solid Feet that shall be desir'd; but if the Sides of the Piece be unequal, find a mean proportional Number, as is before taught, by dividing the Distance upon the Line of Numbers, into two equal Parts: Thus, if the Breadth be 25 Inches, and the Depth 9 Inches, divide the Space upon the Line of Numbers, into two equal Parts, and you will find the middle Point at 15; so is 15 Inches the geometrical mean Proportional sought; then, if you look for 15 upon the Line above mention'd, that 7 Inches, and a little above a half, will be the Length of a Foot.

§ III. *Of unequal squar'd Timber.*

BY unequal squar'd Timber, I mean all such as have unequal Bases; that is, such as is thicker at one End than at the other; and such are most Timber-Trees when they are hewn, and brought to their Squares.

The usual Way to measure such Timber, is to take a Square about the Middle of the Piece, which they take to be a mean Square: This Way, when the Piece is pretty near as thick at one End as at the other, is something near the Truth; but when there is a great Disproportion between the Ends of the Piece, the Error is considerable. All such Solids being the Frustums of Pyramids, the true Way of measuring them must be by Section VII, Chap. II. I shall give an Example or two, which I will work both by the true and false Ways, whereby you will see the Difference.

Example 1. If a Piece of Timber be 25 Inches square at the greater End, and 9 Inches square at the lesser End, and 20 Feet long, how many Feet of Timber is in that Tree?

25
 9
 Sum 34
 half 17 the Side of the Square in the Middle.

$$\begin{array}{r}
 17 \\
 119 \\
 17 \\
 289 \\
 20 \\
 \hline
 144)5780(40.13 \\
 \underline{6200} \\
 580 \\
 \underline{128}
 \end{array}$$

Answer, 40.13 Feet by the false Way.

Y

By

By Rule II, Sect. VII, Chap. II.

25
9

25
9

225

16

16

96

16

3) 256 the Square.

85.3333

225

310.3333

20

144) 6106.6660 (43.101

446

146

266

122

Answer 43.101 Feet, by the true Way; so that there is near 3 Feet Difference.

By Scale and Compasses.

Extend from 1 to 9, that Extent will reach from 25 (the same Way) to 225, the Rectangle of the Sides of the two Bases; then the Difference between the said Sides is 16: Extend from 3 to 16, that Extent will reach from 16 to 85.333, a third Part of the Square; which added to 225, the Sum is 310.333, a mean Area: Then extend from 144 to 310.333, that

Extent

Extent will reach from 20 (the Length) to 43.1 Foot, the Content, the true Way.

Extend the Compasses from 12 to 17, (the Side of the middle Square) that Extent will reach from 20, (the Length, being twice turn'd over) to 40.1 Foot, the Content by the false Way.

Example 2. If a Piece of Timber be 32 Inches broad and 20 Inches deep at the greater End, and 10 Inches broad and 6 deep at the lesser End, and 18 Foot long; how many Feet of Timber are in that Piece?

Rule I, Sect. VII, Chap. II.

$ \begin{array}{r} 32 \\ 20 \\ \hline 640 \\ 60 \\ \hline 38400 \\ 1 \\ \hline 29) 284 \\ 385) 2300 \\ 3909) 37500 \\ 39185) 231900 \\ 391909) 3597500 \\ \hline 144) 5375.754(37.33 \\ \hline 1055 \\ 477 \\ 455 \\ \hline 23 \\ \hline Y. 2 \end{array} $	$ \begin{array}{r} 6 \\ 10 \\ \hline 60 \\ \hline (195.959 \text{ mean Proportional:} \\ 640. \text{ the greater Base,} \\ 60. \text{ the lesser Base,} \\ \hline 895.959 \text{ the Sum.} \\ 6 \frac{1}{3} \text{ Height.} \\ \hline 5375.754 \\ \hline \text{Add} \end{array} $
--	---

$$\text{Add } \left\{ \begin{array}{l} 32 \\ 10 \end{array} \right. \quad \left. \begin{array}{l} 20 \\ 6 \end{array} \right\} \text{Add}$$

$$\text{Sum } 42 \quad 26 \text{ Sum}$$

$$\text{Half } 21 \quad 13 \text{ Half}$$

$$13$$

$$63$$

$$21$$

273 Area in the Middle.

18 Length.

$$2184$$

$$273$$

$$144) 4914 (34.12$$

$$594$$

$$180$$

$$360$$

$$72$$

Feet.

Answer { Content the true Way 37.33
Content the false Way 34.12

By Scale and Compasses.

Extend the Compasses from 1 to 26, that Extent will reach from 32 to 640, the Area of the greater Base.

Then extend from 1 to 10, that Extent will reach from 6 to 60, the Area of the lesser Base : Then extend from 1 to 60, that Extent will reach from 640 to 38400, the Product of the two Areas : Find the square Root thereof, by dividing the Space between 1 and 38400 into two equal Parts, so you will find the middle Point at 195.959, the Root sought ; which is

is a mean Proportional between the greater and lesser Areas: Then add the mean Proportional and two Areas together, and the Sum is 895.959; which multiply'd by 6, (a third Part of the Length) by extending from 1 to 6, that Extent will reach from 895.959 to 5375.75. Then extend from 144 to 5375.75, and that Extent will reach from 1 to 37.33 Feet, the true Content.

For the false Way, half the Sum of the Breadth is 21, which is the Breadth in the Middle; and half the Sum of the Depth is 13: Extend from 1 to 13, that Extent will reach from 21 to 273, the Area of the middle Base: Then extend from 144 to 273, that Extent will reach from 18 (the Length) to 34.12, the Content the false Way.



§ IV. Of round Timber, whose Bases are equal.

THE usual Way to measure round Timber Trees, is to girt them about the Middle with a String, and take the fourth Part of that Girt for the Side of a Square, by which they measure the Piece of Timber as if it was square.

But that this is an Error, I shall make appear as follows. If the Circumference of a Circle be 1, the Area will be .07958; then the fourth Part of 1 is .25, which squar'd makes .0625; this they take for a mean Area, instead of .07958: Therefore the true Content always bears such Proportion to the Content found by the aforesaid customary false Way, as .07958 to .0625; which is nearly as 23 to 18; so that in measuring by that customary false Way, there is above one fifth Part, of what the true Content ought to be, lost.

This Error, tho' it has been so often confuted, yet is it grown so customary in all Places, that there is little Hopes of my prevailing with Men that are so wedded to it, to embrace the Truth; I shall therefore, in the following Examples, shew how to work both the true Way, and also the false or customary Way.

Example 1. If a Piece of Timber be 96 Inches in Circumference or Girth, and 18 Feet long, how many Feet of Timber is contain'd therein?

A fourth Part of 96 is 24

24

—

96

48

—

576 Area Base.

18

—

4608

576

—

144)10368(72

1008

—

288

288

—

...

Or thus.

F. 1.

2 0

2 0

—

4 0

18

—

72 0

Content the false Way 72 Feet.

Then

Then the true Way.

$$\begin{array}{r}
 575 \\
 864 \\
 \hline
 9216 \\
 87958 \\
 \hline
 73728 \\
 46080 \\
 \hline
 82944 \\
 64512 \\
 \hline
 \end{array}$$

733.40928 the Area by Prob. 5. Sect. IX, Chap. H.

$$\begin{array}{r}
 586727424 \\
 73340928 \\
 \hline
 \end{array}$$

$$44)13201.36704(91.67$$

$$\begin{array}{r}
 241 \\
 973 \\
 1096 \\
 \hline
 88
 \end{array}$$

The true Content 91.67 Feet.

By Scale and Compasses.

Extend from 12 to 24, (the fourth Part of the Girth) that Extent turn'd twice over from 18 Feet, (the Length) will at last fall upon 72 Feet, the Content the customary Way.

Extend from 42.54 to 96, (the Girth) that Extent will reach from 18 Feet turn'd twice over, to 91.67 Feet, the true Content.

Example.

Example 2. If a Piece of Timber be 86 Inches Girth, and 20 Feet long, how many Feet are contain'd therein?

The fourth Part of 86 is 21.5

F.	I.	P.	
1	9	6	
1	9	6	
1	9	6	
1	4	1	6
	0	10	9
3	2	6	3
			20
64	2	5	0

The Content the false Way 64.2 Feet.

By

By the true Way.

$$\begin{array}{r}
 86 \\
 86 \\
 \hline
 516 \\
 688 \\
 \hline
 7396 \\
 .07958 \\
 \hline
 59168 \\
 36980 \\
 \hline
 66564 \\
 51772 \\
 \hline
 588.57368 \\
 20 \\
 \hline
 11771.47360 (81.74 \\
 144) \\
 \hline
 251 \\
 1074 \\
 667 \\
 \hline
 91
 \end{array}$$

The true Content 81.74 Feet.

By Scale and Compasses.

Extend from 12 to 21.5, that Extent turn'd twice over from 20, will reach at last to 64.2 Feet, the Content the false Way.

Extend from 42.54 to 86, that Extent turn'd twice over from 20, will at last fall upon 81.74 Feet, the true Content.

These Cylindrical Proportions may be very easily wrought upon the Line of Number.

Problem

Problem 1. Having the Diameter of a Cylinder in Inches, to find the Length of a Foot.

Suppose the Diameter 22.6 Inches.

As 22.6 : to 46.9 :: so is 1 to a fourth Number ; and that to the Length of a Foot in Inches, 4.3.

Extend the Compasses from 22.6 to 46.9, that Extent will reach from 1 to a fourth Number ; then turn them over again, and that will reach to 4.3 Inches.

Problem 2. Having the Diameter in Foot-Measure, to find the Length of a Foot in Foot-Measure.

Suppose the Diameter 1.88 Feet.

Then, as 1.88 : to 1.128 :: so is 1 : to a fourth Number ; and so is that to the Length of a Foot in Foot-Measure .358.

Extend the Compasses from 1.88 to 1.128, that Extent turn'd twice from 1, will reach to .358 Parts of a Foot.

Problem 3. Having the Circumference in Inches, to find the Length of a Foot in Inches.

Suppose the Circumference 71 Inches.

Then, as 71 : to 147.36 :: so is 1 to a fourth Number ; and so is that to the Length of a Foot in Inches 4.3.

Extend the Compasses from 71 to 147.36, that Extent turn'd twice from 1, will reach to 4.3 Inches, the Length of a Foot.

Problem 4. Having the Circumference in Foot-Measure, to find the Length of a Foot in Foot-Measure.

Suppose the Circumference 5.92 Feet.

Then, as 5.92 : to 3.545 :: so is 1 : to a fourth Number ; and so is that to the Length of a Foot in Foot-Measure .358.

Extend

Extend the Compasses from 5.92 to 3.545, that Extent, turn'd twice over from 1, will fall upon 358 Parts of a Foot.

Problem 5. Having the Diameter in Inches, and the Length in Inches, to find the Content in Inches.

Suppose the Diameter 22.6 Inches, and the Length 156 Inches, or 13 Feet.

Then, as 1.128 : to 22.6 :: so is 156 : to a fourth Number ; and so is that to the Content in Inches, 62674.

Extend the Compasses from 1.128 to 22.6, that Extent, turn'd twice from 156, will fall upon 62674 Inches, the Content.

Note, That 1.128 is the Diameter when the Side of the Square equal is 1.

Problem 6. Having the Diameter in Foot-Measure, and Length in Feet, to find the Content in Feet.

Suppose the Diameter 1.88 Feet, and the Length 13 Feet.

Then, as 1.128 : to 1.88 :: so is 13 : to a fourth Number ; and so is that to the Content in Feet, 36.27.

Extend from 1.128 to 1.88, that Extent, turn'd twice from 13, will fall upon 36.27.

Problem 7. Having the Diameter in Inches, and Length in Inches, to find the Content in Feet.

Suppose the Diameter 22.6 Inches, and the Length 156 Inches.

Then, as 46.9 : is to 22.6 :: so is 156 : to a fourth Number ; and so is that to the Content in Feet, 36.27.

Extend from 46.9 to 22.6, that Extent, turn'd twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 46.9 is the Diameter of a Circle, whose Area is 1728.

Problem

Problem 8. Having the Diameter in Inches, and Length in Feet, to find the Content in Feet.

Suppose the Diameter 22.6 Inches, and the Length 13 Feet;

Then, as 13.54 : to 22.6 :: so is 13 to a fourth Number; and so is that to the Content in Feet 36.27.

Extend from 13.54 to 22.6, that Extent, turn'd twice from 13, will fall upon 36.27.

Note, That 13.54 is the Diameter of a Circle, when the Area is 144.

Problem 9. Having the Circumference in Inches, and Length in Inches, to find the Content in Inches.

Suppose the Circumference 71, and the Length 156 Inches;

Then, as 3.545 : is to 71 :: so is 156 to a fourth Number; and so is that to 62674, the Content in Inches.

Extend the Compasses from 3.545 to 71, that Extent, turn'd twice from 156, will fall upon 62674, the Content.

Note, That 3.545 is the Circumference, when the Side of the Square equal is 1.

Problem 10. Having the Circumference in Feet, and Length in Feet, to find the Content in Feet.

Suppose the Circumference 5.92 Feet, and Length 13 Feet;

Then, as 3.545 : to 5.92 :: so is 13 to a fourth Number; and so is that to 36.27.

Extend from 3.545 to 5.92, that Extent, turn'd twice from 13, will fall upon 36.27 Feet, the Content.

Problem 11. Having the Circumference in Inches, and Length in Inches, to find the Content in Feet.

Suppose the Circumference 71 Inches, and Length 156 Inches;

Then, as 147.36 : to 71 :: so is 156 : to a fourth Number ; and so is that to the Content in Feet, 36.27.

Extend the Compasses from 147.36 to 71, that Extent, turn'd twice from 156, will fall upon 36.27 Feet, the Content.

Note, That 147.36 is the Circumference of a Circle whose Area is 1728.

Problem 12. Having the Circumference in Inches, and Length in Feet, to find the Content in Feet.

Suppose the Circumference 71 Inches, and Length 13 Feet ;

Then, as 42.54 : is to 71 :: so is 13 : to a fourth Number ; and so is that to the Content in Feet, 36.27.

Extend the Compasses from 42.54 to 71, that Extent, turn'd twice from 13, will reach to 36.27 Feet, the Content.

Note, That 42.54 is the Circumference of a Circle whose Area is 144.

§ V. Of round Timber, whose Bases are unequal.

THE usual Way to measure round Timber, (as I said before) is to take a fourth Part of the Girth in the Middle of the Piece, for the Side of a mean Square. But this Way I have prov'd to be erroneous in Timber that is all the Way of an equal Thickness; and it must be much more so in Timber that is tapering, and the more tapering it is, the greater is the Error: For to the Error in the last Section, there is added the Error in the third Section; therefore, to measure all such Timber according to Art and Truth, such a Piece ought to be consider'd as a Frustum of a Cone, and should be measur'd by the Rules given in Section VIII, Chapter II, by which Rules the following Examples are wrought.

Example 1. If a Piece of Timber be 9 Inches Diameter at the lesser End, and 36 Inches at the other End, and 24 Feet long, how many Feet of Timber is therein?

$$\begin{array}{r}
 36 \\
 \underline{9} \\
 \text{Rect. } 324
 \end{array}
 \qquad
 \begin{array}{r}
 36 \\
 9 \} \text{Subtract.} \\
 \underline{9} \\
 27 \text{ Difference.} \\
 \underline{27} \\
 189 \\
 \underline{54} \\
 3)729 \text{ the Square.} \\
 \underline{243} \text{ one third.} \\
 324 \text{ Rectangle add.} \\
 \underline{\hspace{1cm}} \\
 567
 \end{array}$$

$$\begin{array}{r}
 .7854 \\
 567 \\
 \hline
 54978 \\
 47124 \\
 39270 \\
 \hline
 \text{A mean Area } 445.3218 \\
 24 \\
 \hline
 17812872 \\
 8906436 \\
 \hline
 144)10687.7232(74.22 \\
 \hline
 607 \\
 317 \\
 292 \\
 \hline
 4
 \end{array}$$

Answer, 74.22 Feet.

Or thus, by Feet and Inches.

F. I.	F. I.	
3 0	2 3	Difference.
0 9	2 3	
2 03 Rect.	4 6	
	0 6 9	
	5 0 9	the Square.
	1 8 3	one Third.
	2 3 0	Rectangle added.
	3 11 3	a mean Square.

Z 2

Then,

F. I. P.

Then, as 14 : to 11 :: so is 3 11 3 to the Area.

11

$$\begin{array}{r}
 \hline
 7 \overline{) 43 \ 3 \ 9} \\
 \hline
 2 \overline{) 6 \ 2 \ 3} \\
 \hline
 3 \ 1 \ 1 \ 6 \\
 \hline
 18 \ 6 \ 9 \ 0 \\
 \hline
 74 \ 3 \ 0 \ 0
 \end{array}$$

Here, instead of dividing by 14, I divide by 7 and by 2, because twice 7 is 14.

And instead of multiplying by 24 Feet, the Length, I multiply by 6 and by 4, because 6 times 4 is 24.

By Scale and Compasses this is too troublesome.

Example 2. If a Piece of Timber be 136 Inches Circumference at one End, and 32 Inches Circumference at the other End, and 21 Feet long, how many Feet of Timber is contain'd in that Piece?

136
32

136
32

272
408

104
104

Difference.

4352

416

104

3)10816 the Square.

3605.333 one Third.

4352 Rectangle add.

7957.333 a mean Circumf. squar'd.

.07958

63658664

39786665

71615997

55701331

633.24456014 the mean Area.

21

63324456014

126648912028

13298.13576294

144)13298.13(92.34

338

501

693

117

Answer, 92.34 Feet.

By Feet and Inches, thus;

F. I.
22 4
2 8

22 8
7 6 8

30 2 8

F. I.
8 8 Difference.
8 8

69 4
5 9 4

3)75 1 4 the Square.

25 0 5 4
30 2 8 0

55 3 1 4 the Sq. of the Circumf.

F. I. P. S.
As 88 : to 7 :: 55 3 1 4 : to the mean Area.
7

11)386 9 9 4

8) 35 1 11 9

4 4 8 11 the mean Area.

7
30 9 2 5
3

Facit. 92 3 7 3

§ VI. *Of the five regular Bodies.*

THESE Bodies may all be measur'd by the 4th Section of Chap. II, except it be the Cube, or Hexaedron, which is already measur'd in Section I of that Chapter.

I. *Of the TETRAEDRON.*

A Tetraedon is a Solid, contain'd under four equal and equilateral Triangles.

Let ABCD be a Tetraedron, whose Side is 12 Inches, the perpendicular Height 9.798 Inches.



By Sect. V, Chap. I, the Area of the Triangle will be found 62.352; a third Part of it is 20.784; which multiply'd by the perpendicular Height, the Product is 203.641632 solid Inches, the Content.

10.392 the Perpendicular of the Triangle.
6 half the Side.

62.352 Area of the Triangle:

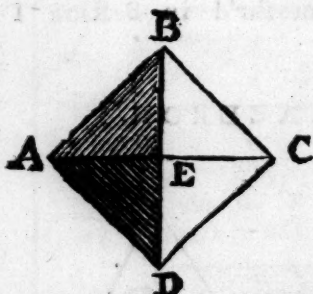
20.784 one third Part.
9.798 the perpendicular Height.

166272
187056
145488
187056

203.641632 the Solidity.

The superficial Content is four times the Area of the Triangle, viz. 249.408 Inches, because there is 4 Triangles.

2. Of the OCTAEDRON.



The Octaedron is a Body contain'd under eight equal and equilateral Triangles.

Let ABCDE be an Octaedron, whose Side is 12 Inches; the Content solid and superficial is requir'd.

An Octaedron is compos'd of two quadrangular Pyramids join'd together by their Bases; therefore, if the Area of the Base be multiply'd into a third Part of the Length of both Pyramids, the Product will be the solid Content.

5.6568 a third Part of the Length.

144 Area of the square Base.

226272
226272
56568

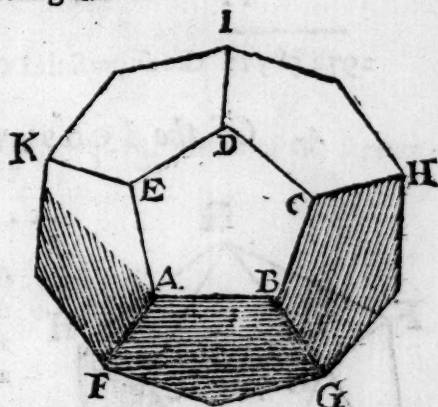
8145792 the Solidity.

The superficial Content will be just double to that of the Tetraedron, viz. 498.816, because the Side of this is suppos'd to be equal to the Side of that, and because the Octaedron is contain'd under eight Triangles, and the Tetraedron but under four.

3. Of the DODECAEDRON.

THE Dodecaedron is a solid Body, contain'd under twelve pentangular Plains.

Let ABCDEFGHIK be a Dodecaedron, each Side thereof being 12 Inches; the Content solid and superficial is required.



The Solidity of the Dodecaedron is compos'd of 12 pentangled Pyramids, whose Vertexes all meet in the Center. Therefore, if we find the Solidity of one of those Pyramids, and multiply that by 12, that Product will be the Solidity of the Dodecaedron.

The Altitude of one of the pentangled Pyramids will be found to be 13.36219.

The Perpendicular of the Pentagon will be

8.258292

30 half Sum of the Sides.

247.748760 Area of the Pentagon.

60454.4 a third Part of 13.36219 inverted.

99099504

9909950

1238744

99099

1486

1103.48783 Content of one Pyramid:

12

13241.85396 the Solidity of the Dodecaedron.

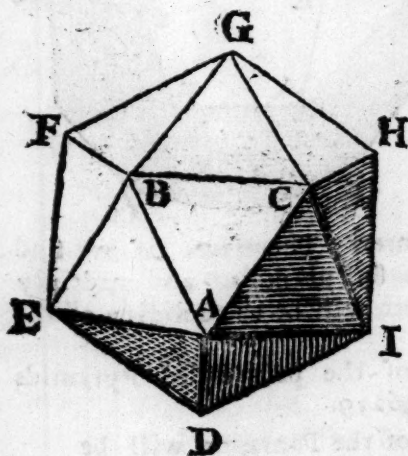
If

If the Area of the Pentagon be multiply'd by 12, the Product will be the superficial Content.

$$\begin{array}{r} 247.74876 \\ 12 \\ \hline \end{array}$$

2972.98512 the superficial Content.

4. Of the ICOSAEDRON.



The Icosaedron is a solid Body, contain'd under twenty equal and equilateral Triangles.

Let ABCDEFGHI be an Icosaedron, each Side thereof being 12 Inches; the Content solid and superficial is requir'd.

The Icosaedron is compos'd of twenty triangular Pyramids, with their Vertexes all joining in the Center.

Therefore, if the solid Content of one Pyramid be multiply'd by 20, the Product is the whole solid Content of the Icosaedron.

Chap. IV. *the five regular Bodies.* 263

10.39224 the Perpendicular of the Triangle.
6 half the Side.

62.35344
20

1247.06880

3.0230456 the third Part of the Altit. of the Pyram.
44353.26

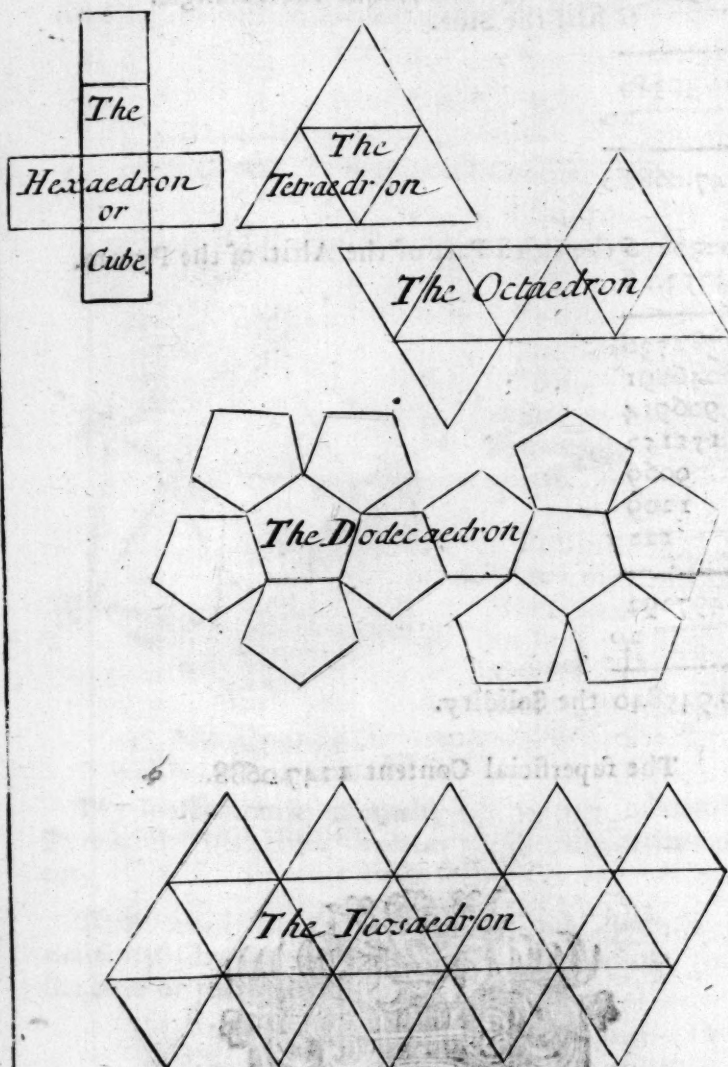
181382736
6046091
906914
151152
9069
1209
121

188.497292
20

3769.945840 the Solidity.

The superficial Content 1247.0688.





By these Figures you may cut these Bodies in fine
 Past-board, cutting all the Lines half through, and so
 turn them up and glew them. A

A TABLE shewing the Solidity and superficial Content of any of the regular Bodies, the Side being 1, or Unity.

The Names of the Bo- dies.	The Solidity.	Superficies
Tetraedron	0.1178511	1.732051
Octaedron	0.4714045	3.464102
Hexaedron	1.0000000	6.0000000
Icosaedron	2.181695	8.660254
Dodecaedron	7.663119	20.645729

By this Table, the Content, either superficial or solid, of any of these Bodies, may very readily be found; for all like superficial Figures, are in Proportion one to another, as are the Squares of their like Sides: Therefore it will be, as the Square of 1 (which is 1) : is to the superficial Content in the Table :: so is the Square of the Side of the like Body : to the superficial Content of the same Body. Therefore, if the Number in the Table be multiply'd by the Square of the Side given, the Product is the superficial Content requir'd.

Again, all like Solids are in such Proportion to each other, as are the Cubes of their like Sides. Therefore it will be, as 1 : (which is the Cube of 1) is to the solid Content in the Table :: so is the Cube of the Side given : to the solid Content requir'd. Therefore, if the Number in the Table be multiply'd by the Cube of the given Side, the Product will be the solid Content of the same Body.

Example 1. If the Side of a Dodecaedron be 12 Inches, (as before) what is the Content solid and superficial?

7.6631119 the tabular Number.

1728 the Cube of the Side.

61304952

15326238

53641833

7663119

13241.869632 the solid Content, nearly the same as (before).

20.645729 the tabular Number.

144 the Square of the Side.

82582916

82582916

20645729

2972.984976 the superficial Content.

By Scale and Compasses.

Extend from 1 to 12, (the Side) that Extent, being turn'd three times over from 7.663119, will at last fall upon 13241.86, &c. the solid Content.

And if you apply the same Extent twice from 20.645729, it will at last fall upon 2972.98, &c. the superficial Content.

Example

Example 2. If the Side of an Octaedron be 20 Inches, what is the Content solid and superficial?

.4714045 the tabular Number.

8000 the Cube of the Side.

3771.2360000 the solid Content.

3.464102 the tabular Number.

400 the Square of the Side.

1385.640800 the superficial Content.

By Scale and Compasses.

Extend from 1 to 20, that Extent, turn'd three times over from .4714045, will at last fall upon 3771.236, the solid Content. The same Extent, turn'd twice over from 3.464, &c. will at last fall upon 1385.64, the superficial Content.

§ VII. *How to measure any irregular Solid.*

IF you have any Piece of Wood or Stone that is craggy and uneven, and you desire to find the Solidity, put the Solid into any regular Vessel, as a Tub, a Cistern, or the like, and pour in as much Water as will just cover it; then take out the Solid, and measure how much the Fall of the Water is, and so find the Solidity of that Part of the Vessel.

Example. Suppose a Piece of Wood or Stone to be measur'd, and suppose a Tub 32 Inches Diameter, into which let the Stone or Wood be put, and cover'd with Water; then, when the Solid is taken out, suppose the Fall of the Water 14 Inches, square 32, and multiply the Square by .7854, and the Product will be 804.2496, the Area of the Base; which multiply'd by 14, the Depth or Fall of the Water, and the Product is 11259.49, &c. which divided by 1728, the Quotient is 6.51 Feet; and so much is the solid Content requir'd.





C H A P. V.

Practical Questions in MEASURING.

Question 1. IF a Pavement be 47 Feet 9 Inches long, and 18 Feet 6 Inches broad, I demand how many Yards are contain'd therein?

F.	I.	
47	9	47.75
18	6	18.5
376		23875
47		38200
23	10 6	4775
9	0 0	
4	6 0	9)883.375
883	4 6	98 1

Answer, 98 Yards 1 Foot.

Quest. 2. There is a Room, whose Length is 215 Feet, and the Breadth 17.5 Feet, is to be pav'd with Stones, each 18 Inches square; I demand how many such Stones will pave it?

21.5	1.5
<u>17.5</u>	<u>1.5</u>
1075	75
1505	15
<u>215</u>	
2.25) 376.25 (167	2.25 Area of one Stone.
<u>1512</u>	
<u>1625</u>	
50	

Answer 167 Stones.

Quest. 3. There is a Room 109 Feet 9 Inches about, and 9 Feet 3 Inches high, which is all (except two Windows, each 6 Feet 6 Inches high, and 5 Feet 9 Inches broad) to be hung with Tapestry that is Ell broad; I desire to know how many Yards will hang the said Room?

From the Content of the Room subtract the Content of the Windows, and divide the Remainder by the square Feet in an Ell.

3.75	109.75 Length.	5.75
<u>3</u>	9.25 Breadth.	<u>6.5</u>
11.25	54875	2875
	21950	3450
	<u>98775</u>	
	1015.1875 Content of the Room.	37.375
	74.75 Content of the Windows sub.	<u>2</u>
		74.750
11.25	940.4375 (83.59	

4043
6687
10625

500

Answer 83.59 Yards.

Quest. 4. If the Axis of a Globe be 27.5 Inches,
I demand the Content solid and superficial?

3.1416
27.5

157080
219912
62832

86.39400 The Circumference.

86.394

86.394 the Circumference.

27.5 the Diameter.

$$\begin{array}{r}
 431970 \\
 604758 \\
 \hline
 172788
 \end{array}$$

6) 2375.8350 the Superficial Content.

395.9725 a sixth Part.

27.5

$$\begin{array}{r}
 19798625 \\
 27718075 \\
 \hline
 7919450
 \end{array}$$

10889.24375 the Solidity in Inches.

Answer { 6.3 Feet solid.
16.49 Feet superficial.

Quest. 5. There is the Frustum of a Globe, the Diameter of whose Base is 24 Inches, and the Altitude thereof is 10 Inches; what is the Content solid and superficial?

Find the Superficies as is directed in Pag. 188, and find the Solidity by the first Theorem in Pag. 190.

24	.7854	.7854	20
24	576	400	20
—	47124	314.1600	400
96	54978		
48	39270		
—			
576	452.3904	} add.	
	314.16		
	—		

766.5504 the Curve Superficies.

452.3904 the Base add.

1218.9408 { The whole superficial Content in Inches.

$$12 \times 12 = 144$$

3

432

100 the Square of the Alt. add.

532 the Sum.

10 multiply by the Alt.

5320

.5236 multiply.

31920

15960

10640

26600

2785.5520 the Solidity in Inches.

Quest. 6. If a Tree girt 18 Feet 6 Inches, and be 24 Feet long; how many Tuns of Timber are contain'd in that Tree?

F. I.

4) 18 6 the Girth.

4 7 6 a 4th Part.

4 7 6

18 6 0

2 8 4 6

2 3 9

21 4 8 3

6

128 4 1 6

4

4) 0) 51 3 4 6 0

12 33

Answer, 12 Tuns 33 Feet 4 Inches 6 Parts.

Note,

Here I multiply by 6 and by 4, because 6 times 4 is 24.

Note, That 40 Feet of Timber is a Tun, and 50 Feet a Load.

Note also, That 4 Feet broad, 4 Feet deep, and 8 Feet long, is a Chord of Fire-Wood, that is 128 Cubical Feet.

Quest. 7. There is a Cellar to be dug by the Floor, whose Length is 33 Feet 7 Inches, and the Breadth is 18 Feet 9 Inches, and its Depth is to be 5 Feet 9 Inches, I demand how many Floors of Earth are in that Cellar ?

F.	I.	
33	7	the Length.
18	9	the Breadth.
264		
33		
16	9	6
8	4	9
9	0	0
1	6	0
629	8	3
5	9	the Depth.
3148	5	3
314	10	1
157	5	0
324)	3620	8 4(11
	380	
	56	

Answer, 11 Floors 56 Feet.

Note, That 18 Feet square, and a Foot deep, is a Floor of Earth, that is, 324 solid Feet.

Quest.

Quest. 8. There is a Roof cover'd with Tiles, whose Depth on both Sides (with the usual Allowance at the Eaves) is 35 Feet 6 Inches, and the Length 48 Feet 9 Inches; how many Squares of Tiling are contain'd therein?

F.	I.	
48	9	
35	6	
240		
144		
24	4	6
17	6	0
8	9	0
17	30	7 6

Answer, 17 Squares 30 Feet.

Quest. 9. There is a Cone, whose Diameter at the Base is 42 Inches, and the perpendicular Height 94 Inches, and it is requir'd to cut off two solid Feet from the top End thereof; I demand what Length upon the Perpendicular must be cut off?

$$\begin{array}{r}
 42 \quad 1728 \quad 94 \\
 42 \quad 2 \quad 94 \\
 \hline
 84 \quad 3456 \quad 376 \\
 168 \quad 846. \\
 \hline
 1764 \quad 8836 \text{ Square.} \\
 .7854 \quad 94 \\
 \hline
 7056 \quad 35344 \\
 8820 \quad 79524 \\
 \hline
 14112 \\
 12348 \quad 830584 \text{ the Cube.} \\
 \hline
 1385.4456 \\
 94 \\
 \hline
 55417824 \\
 124690104 \\
 \hline
 3) 130231.8864 \\
 \hline
 43410.6288
 \end{array}$$

All like Bodies are in triplicate Reason of their homologous Sides by *Emc.* 12, 12; 12, 18; and 11, 33; therefore it will be,

Solidity of the Cone. Cube Alt. Solidity of 2 Feet.

As 43410.6288 : 830584 :: 3456 :

$$\begin{array}{r}
 3456 \\
 4983504 \\
 4152920 \\
 3322336 \\
 2491752 \\
 \hline
 43410.6288) 2870498304 (66124 \text{ the Cube of the} \\
 \text{Length.} \\
 \hline
 265860576 \\
 5396803 \\
 1055740 \\
 187528
 \end{array}$$

$$\begin{array}{r} 66124(40.43 \\ 64 \end{array}$$

2124000 Resolvend.

$$\begin{array}{r} 12 \\ 48 \end{array}$$

492 Divisor.

$$\begin{array}{r} 120 \\ 4800 \end{array}$$

48120 Divisor.

$$\begin{array}{r} 64 \\ 1920 \\ 19200 \end{array}$$

1939264 Subtrahend.

184736000 Resolvend.

$$\begin{array}{r} 1212 \\ 489648 \end{array}$$

4897692 Divisor.

$$\begin{array}{r} 27 \\ 10908 \\ 1468944 \end{array}$$

147003507 Subtrahend.

37732493 Remains.

Answer. The Length upon the Perpendicular must be 40.43 Inches. If it had been 3 Feet, the Length had been 46.29 Inches.

Quest. 10. If a square Piece of Timber be 12 Feet long, and if the Side of the Square of the greater Base be 21 Inches, and the Side of the Square of the lesser Base be 3 Inches; how far must I measure from the greater End, to cut off 5 solid Feet?

First, find the Length of the whole Pyramid thus; the Difference between 21 and 3 is 18; then,

$$\begin{array}{l} \text{Diff. Length.} \quad \text{great. Length.} \\ \text{As } 18 : 12 :: 21 : 14, \end{array}$$

So I find the whole Length of the Pyramid 14 Feet, or 168 Inches.

The solid Content of the whole Pyramid is 24696 Inches, and the solid Content of 5 Feet is 8640; which subtracted from 24696, there remains 16056 Inches. Then, the Cube of 168 (the Length) is 4741632. Then,

$$\text{As } 24696 : 4741632 :: 16056 :$$

To 3082752, whose Cube Root is 145.74; subtract this Root from 168, (the Length) and there remains 22.46 Inches, which is the Length of 5 solid Feet at the great End.

Quest. 11. Three Men bought a Grinding-Stone of 40 Inches Diameter, which cost 20 Shillings; of which Sum, the first Man paid 9 Shillings, the second 6 Shillings, and the third 5 Shillings; I demand how much of the Stone each Man must grind down, proportionable to the Money he paid?

All Circles are in duplicate Reason of their Diameters, by *Euc.* 12, 2.

Square the Semidiameter, which makes 400. Then,

$$\begin{array}{l} \text{s. Square.} \quad \text{s.} \\ \text{As } 20 : 400 :: 9 : 180. \end{array}$$

This 180 is the Square of the Semidiameter of the Circle belonging to the first Man.

A a 2

And,

s. s.

And, as 20 : 400 :: 6 : 120.

This 120 is the Square of the Semidiameter of the Circle belonging to the second.

s. s.

And, as 20 : 400 :: 5 : 100.

This 100 is the Square of the Semidiameter of the Circle belonging to the third.

Then, from 400 (the Square of the Semidiameter of the Stone) subtract 180, and there remains 220, whose square Root is 14.83 Inches ; which subtracted from 20 Inches, (the Semidiameter) there remains 5.17 Inches, which is the Breadth of the Ring, or Part of the Stone which must be ground down by the first.

Then, from 220 subtract 120, and there remains 100, whose square Root is 10 ; subtract that from 14.83, and there remains 4.83 Inches, the Breadth of the Ring, or Part to be ground down by the second Man. The third must grind down the Remainder, which is 10 Inches, the square Root of 100.

This Question may very easily and speedily be perform'd Geometrically, as in the annexed Scheme.

First,

Quest. 13. There is a Cistern, whose Depth is seven Tenths of the Width, and the Length is six times the Depth, and the solid Capacity is 367.5 Feet. I demand the Depth, Width, and Length, and how many Bushels of Corn it will hold ?

First, you must find three Numbers, in Proportion to the Depth, Width, and Length; thus, suppose the Depth 7, then the Width will be 10, and the Length 42; which multiply'd together, the Product is 2940, which is the solid Inches in a Cistern, whose Depth is 7, Width 10, and Length 42. But the solid Inches in the Question are 635040 ($= 367.5 \times 1728$), then the Cube of the suppos'd Width is 1000. So it will be,

As 2940 : 1000 :: 635040 : 216000, whose Cube Root is 60, which is the true Width; 7 Tenths thereof is 42, the Depth; and 6 times 42 is 252 Inches, the Length; which three Numbers being multiply'd together, the Product will be 635040. If these solid Inches be divided by 2150.42, and the Quotient is $295\frac{66\frac{1}{2}}{2150.42}$ Bushels, or 36 Quarters, 7 Bushels, 1 Peck, 4 Pints. And so much will the Cistern hold.

Quest. 14. Suppose, Sir, a Bushel be exactly round, Whose Depth being measur'd, 8 Inches is found, If the Breadth 18 Inches and an half you discover, This Bushel is legal all *England* over. But a Workman would make one of another Frame; Sev'n Inch and a half must be the Depth of the same: Now, Sir, of what Length must the Diameter be, That it may with the former in Measure agree ?

Ques 13. There is a Cistern, whose Depth is 18.5
even Tenths of the Width, and the Length is 58.1
the Depth, and the solid Capacity is 37 Feet. 1
demand the Depth, Width, and Length, and how
many Bushels of Corn it will hold?
1480
581

First, you must find three Numbers, whose Proportion
to the Depth, Width, and Length, shall be as the
Depth 7, then the Width will be 10, and Length
42; which multiply'd together, the Product is 2940;
which is the solid Inches in a Cistern, whose Depth
is 7, Width 10, and Length 42. But 58.1 in
lies in the Question is 2940 (= 36008328)
then the Cube of the supposed Width is 1000
239575

will be. In the Table of Numbers, find 1000
As 2940 : 1000 :: 2940 : 10
Cube Root is 10, which is the true Width: 7 Tenths
thereof is 7, which is the true Depth: 42 Tenths
thereof is 4.2, which is the true Length: which three Numbers being mul-

2150.425200 the solid Inches in a Bushel.
7.5)2150.4252(286.72336

650
504

542
175
252
270
450

0

$$\begin{array}{r}
 .7854)286.72336 \\
 \hline
 51103 \\
 39793 \\
 52360 \\
 5236 \\
 \hline
 29)265 \\
 261 \\
 \hline
 381)406 \\
 381 \\
 \hline
 38207)256600
 \end{array}$$

Answer, The Diameter must be 19.107 Inches, if the Depth be 7.5 Inches.

Quest. 15. In the Midst of a Meadow well stored with Grass,
I took just an Acre to tether my As;
How long must the Cord be, that feeding all round,
He mayn't graze less nor more than his Acre of Ground?

By Problem 10, Section IX, Chap. I, find the Diameter of a Circle containing an Acre; half that will be the Length of the Cord.

The Work.

660 Feet, the Length of an Acre.

66 Feet, the Breadth of an Acre.

$$\begin{array}{r}
 3960 \\
 3960 \\
 \hline
 \hline
 \end{array}$$

43560 the square Feet in an Acre.

As

As 1 : 1.2732 :: 43560

43560

763920

63660

38196

50928

55460.5920 (235.5 Diameter)

4

117.75 half.

43)154

129

465)2560

2325

4705)23559

23525

34

Answer, The Cord must be 117 Feet and 9 Inches.

Quest. 16.

Quest. 16. A Malster has a Kiln that is 16 Feet 9 Inches square; but he is minded to pull it down, and build a new one, that may be big enough to dry three times as much at a Time as the old one will do; I demand how much square the New one must be?

$$\begin{array}{r}
 16.5 \\
 16.5 \\
 \hline
 825 \\
 990 \\
 165 \\
 \hline
 272.25 \text{ the Area of the old one.} \\
 3 \\
 \hline
 816.75 (28.57) \\
 4 \\
 \hline
 48)416 \\
 384 \\
 \hline
 565)3275 \\
 2825 \\
 \hline
 5707)45000 \\
 39949 \\
 \hline
 5051
 \end{array}$$

Answer, The Side of the new one must be 28 Feet, and near 7 Inches.

Quest. 17.

Quest. 17. If a round Cistern be 26.3 Inches Diameter, and 52.5 Inches deep; how many Inches Diameter must a Cistern be, to hold twice the Quantity, the Depth being the same? And how many Ale-Gallons will each Cistern hold?

$$\begin{array}{r} 26.3 \\ 26.3 \\ \hline 789 \\ 1578 \\ 526 \\ \hline \end{array}$$

691.69 the Square.

2

$$\begin{array}{r} 1383.38(37.19 \\ 9 \\ \hline \end{array}$$

$$\begin{array}{r} 67)483 \\ 469 \\ \hline \end{array}$$

$$\begin{array}{r} 741)1438 \\ 741 \\ \hline \end{array}$$

$$\begin{array}{r} 7429)69700 \\ 66861 \\ \hline \end{array}$$

$$2839$$

The Diameter of the greater is 37.19 Inches.

691.69 the Square of the lesser Cistern's Diam.

.7854

276676

345845

553352

484183

543.253326 Area of the Base

52.5

2716266630

1086506652

2716266630

28520.7996150 solid Content in Inches.

282)28520.799(101.137 Gallons.

320

387

1059

2139

165

Note, That 282 solid Inches is an Ale or Beer Gallon, and 231 a Wine Gallon.

And 359.05 is the Square of the Diameter of a Circle that will hold a Gallon of Ale, at an Inch deep, and 294.12 for Wine.

You may find the Content in Gallons, thus: Divide the Square of the Diameter by 359.05, and multiply the Quotient by the Depth.

$$\begin{array}{r}
 359.05 \overline{) 1383.38} \quad (3.853 \\
 \underline{113715} \\
 246230 \\
 \underline{18990} \\
 56340 \\
 \underline{55870} \\
 470
 \end{array}$$

The Content of the greater 202.2825 Gallons.

Quest. 18. If the Diameter of a Cask at the Bung be 32 Inches, and at the Head 25 Inches, and the Length 40 Inches, how many Ale Gallons is contain'd therein?

$$\begin{array}{r}
 25 \quad 32 \quad 359 \\
 25 \quad 32 \quad 3 \\
 \hline
 125 \quad 64 \quad 1077 \\
 50 \quad 96 \\
 \hline
 625
 \end{array}$$

1024 Square of the Bung Diam.

1024 the same.

625 Square of the Head Diameter.

$$1077 \overline{) 2673} \quad (2.48$$

40

5190

8820 99.20

204

Answer, 99.2 Gallons.

Otherwise, You may find a mean Diameter, and work by Scale and Compasses, thus; subtract 25 from 32, and there remains 7, which multiply by .7, the Product is 4.9, which added to 25, the Sum is 29.9. Then, extend the Compasses from 18.95 to 29.9, that Extent, turn'd twice from 40, (the Length) will fall upon 99.6 Gallons; something more than before.

Quest. 19. There is a Stone 20 Inches long, 15 Inches broad, and 8 Inches thick, which weighs 217 Pounds; I demand the Length, Breadth, and Thickness of another of the same Kind and Shape, which weighs 1000 Pounds?

The Cube of 20 (the Length) is 8000. Then, (by *Euc. 11. 33.*)

As 217 : 8000 :: 1000 : 36870.645, whose Cube Root is 33.28 Inches, the Length of the Stone weighing 1000 Pounds. Then say,

$$\text{As } 20 : 33.28 :: 15 : 24.96$$

$$\text{As } 20 : 33.28 :: 8 : 13.312$$

Answer, $\left\{ \begin{array}{l} \text{The Length—} 33.28 \\ \text{The Breadth—} 24.96 \\ \text{The Thickness—} 13.312 \end{array} \right\}$ Inches.

Quest. 20. If an Iron-Bullet, whose Diameter is 4 Inches, weighs 9 Pounds, what will be the Weight of another Bullet, (of the same Metal) whose Diameter is 9 Inches?

The Cube of 4 is 64, and the Cube of 9 is 729: Then, (by *Euc. 12. 18.*)

lb

lb

$$\text{As } 64 : 9 :: 729 : 102.515$$

lb oz. dr.

Answer, It weighs 102 8 4 ferè.

Quest. 21.

Quest. 21. There is a square Pyramid of Marble, each Side of its Base is 5 Inches, and the Height thereof 15 Inches, and its Weight is 12 Pounds and a Quarter; I demand the Weight of another like square Pyramid, each Side of whose Base is 30 Inches?

The Cube of 5 is 125, and the Cube of 30 is 27000.
Then, (by *Euc. 12. 12.*)

$$\begin{array}{ccc} & \text{lb} & \text{lb} \\ \text{As } 125 : 12.25 :: 27000 : 2646 \end{array}$$

Answer, The Weight is 2646 Pounds.

Quest. 22. There is a Ball or Globe of Marble, whose Diameter is 6 Inches, and its Weight 11 Pounds; what will be the Diameter of another Globe of the same Marble, that weighs 500 Pounds?

The Cube of 6 is 216. Then,

$$\begin{array}{ccc} & \text{lb} & \text{lb} \\ \text{As } 11 : 216 :: 500 : 9818.1818 \end{array}$$

Whose Cube Root is 21.4 Inches, the Diameter sought;

Quest. 23. There is a Frustum of a Pyramid, whose Bases are regular Octagons; each Side of the greater Base is 21 Inches, and each Side of the lesser Base is 9 Inches, and its Length is 15 Feet; I demand how many solid Feet are contain'd therein?

C 0 2

4.8284

4.8284 the tabular Number, Page 164.

237 the Square of a mean Side.

337988	21	12
144852	19	12
96568		
11443308	189	3)144
15	48	
57216540	237	48
11443308		
144)17164.9620(119.2		
276		
1324		
289		
1		

Answer, 119.2 solid Feet.

Quest. 24. There is a Frustum of a Cone, the Diameter of the greater Base is 36 Inches, and the Diameter of the lesser Base is 20 Inches, and the Length, or Height, is 215 Inches; I demand the Length and solid Content of the whole Cone, and also the solid Content of the given Frustum?

First, find the Length of the whole Cone; thus,

From 36
subtr. 20

As 16 : 215 :: 36 : 483.75

So the Length of the whole Cone is $483\frac{3}{4}$ Inches.

Then

Then find the Content of the whole Cone.

36	1017.8784
36	52.161
216	10178784
108	6107270
1296	101788
.7854	20357
	5089
	Feet.
5184	1728)164132.88(94.98
6480	
10368	8612
9072	17008
	14568
Area Base 1017.8784	
	744

Thus I find the Solidity of the whole Cone 94.98 Feet.

Then find the solid Content of the top Part that is wanting;

C c 3

.7854

.7854 the Area of Unity.

400 the Square of 20.

3)314.1600 Area of the lesser Base.

104.72

a third Part.

268.75

Altitude of the top Part.

52360

73304

83776

62832

20944

2728)28143.5000(16.28 Feet.

10863

4955

14990

1166

Feet.

Content of the whole 94.98

Content of the top Piece 16.28

Content of the Frustum 78.7

Quest. 25. If the top Part of a Cone contains 26171 solid Inches, and 200 Inches its Length, and the lower Frustum thereof contains 159610 solid Inches; I demand the Length of the whole Cone, and the Diameter of each Base?

200

200

159610 } add.
26171

40000

185781 the Sum.

200

800000

As 26171 : 8000000 :: 185781 : to 56789881,
whose Cube Root is 384.3 Inches, the Length of the
whole Cone.

Then find the Diameter of the lesser Base; thus,

200)26171

130.855

3

392.565 Area of the lesser Base.

Then, by Prob. 10, Sect. IX, Chap. I.

As 1 : 1.2732 :: 392.565

1.2732

785130

1177695

2747955

785130

392565

499.8137580(22.35

4

42)99

84

443)1581

1329

4465)25237

22325

2912

Again,

Lesser Leng. Less. Diam. Greater Leng. Gr. Diam.
 Again, as 200 : 22.35 :: 384.3 : 42.94

Inches

Answer, { The Length of the whole Cone — 384.3
 { The Diameter of the greater Base — 42.94
 { The Diameter of the lesser Base — 22.35

Quest. 26. There is a Frustum of a Cone, whose solid Content is 20 Feet, and its Length 12 Feet; and the greater Diameter bears such Proportion to the lesser as 5 to 2; I demand the Diameters?

$$\begin{array}{rcl} 5 \times 5 & = & 25 \\ 2 \times 2 & = & 4 \\ 5 \times 2 & = & 10 \\ \hline & & 39 \end{array} \qquad \begin{array}{r} 3)12 \\ \hline 4)20(5 \text{ Feet.} \end{array}$$

The Sum 39

These 5 Feet are the triple of a mean Area.

Then, as 1 : 1.27324 :: 5 : 6.3662

So the triple Square of a mean Diameter is 6.3662.

Then, as 39 : 6.3662 :: 25 : 4.080897.

This 4.080897 is the Square of the greater Diameter, whose square Root is 2.020123 Feet, which is 24.24147 Inches. Then,

As 5 : 24.24147 :: 2 : 9.69659

So the greater Diameter is 24.24147, and the lesser Diameter is 9.69659 Inches.

Quest. 27. There is a Room of Wainscot 129 Feet 6 Inches in Circumference, and 16 Feet 9 Inches high, (being girt over the Mouldings); there are two Windows, each 7 Feet 3 Inches high, and the Breadth of each, from Cheek to Cheek, 5 Feet 6 Inches; the Breadth of the Shutters of each is 4 Feet 6 Inches; the Cheek-Boards and Top and Bottom Boards of each Window,

Window, taken together, is 24 Feet 6 Inches, and their Breadth 1 Foot 9 Inches; the Door-Case 7 Feet high, and 3 Feet 6 Inches wide; the Door 3 Feet 3 Inches wide. I demand how many Yards of Wainscot are contain'd in that Room?

$$\begin{array}{r} \text{F. I.} \\ 129 \quad 6 \\ 16 \quad 9 \end{array}$$

$$\begin{array}{r} 782 \quad 0 \\ 129 \quad 6 \\ 64 \quad 9 \\ 32 \quad 4 \quad 6 \\ \hline 2169 \quad 1 \quad 6 \end{array}$$

$$\begin{array}{r} \text{F. I.} \\ 7 \quad 3 \\ 4 \quad 6 \end{array}$$

$$\begin{array}{r} 29 \quad 0 \\ 3 \quad 7 \quad 6 \\ \hline 32 \quad 7 \quad 6 \\ 16 \quad 3 \quad 9 \text{ Half.} \\ \hline 48 \quad 11 \quad 3 \\ 2 \\ \hline 97 \quad 10 \quad 6 \end{array}$$

$$\begin{array}{r} \text{F. I.} \\ 3 \quad 3 \\ 7 \quad 0 \end{array}$$

$$\begin{array}{r} 22 \quad 9 \\ 11 \quad 4 \quad 6 \text{ Half.} \\ \hline 34 \quad 1 \quad 6 \end{array}$$

$$\begin{array}{r} \text{F. I.} \\ 7 \quad 3 \\ 5 \quad 6 \end{array}$$

$$\begin{array}{r} 36 \quad 3 \\ 3 \quad 7 \quad 6 \\ \hline 39 \quad 10 \quad 6 \\ 2 \\ \hline 79 \quad 9 \quad 0 \end{array}$$

$$\begin{array}{r} \text{F. I.} \\ 24 \quad 6 \\ 1 \quad 9 \end{array}$$

$$\begin{array}{r} 24 \quad 6 \\ 18 \quad 4 \quad 6 \\ \hline 42 \quad 10 \quad 6 \\ 2 \end{array}$$

$$85 \quad 9 \quad 0$$

$$\begin{array}{r} \text{F. I.} \\ 3 \quad 6 \\ 7 \quad 0 \end{array}$$

$$\begin{array}{r} 24 \quad 6 \\ 79 \quad 9 \\ \hline 104 \quad 3 \end{array}$$

add.

The Content of the Room	2169	1	6
The Shutters, at Work and half	97	10	6
The Door, at Work and half	34	1	6
The Cheek-Boards, &c.	85	9	0

The Sum	2386	10	6
The Window-Lights and Door-Cafe deduct	104	3	0

9) 2282 7 6

253 5

Answer, 253 Yards 5 Feet:

Quest. 28. There is a Wall which contains 18225 Cube Feet, and the Height is 5 times the Breadth, and the Length 8 times the Height; what is the Length, Breadth, and Height?

Suppose the Breadth 2, then the Height must be 10, and the Length 80; which three Numbers multiply'd together, the Product will be 1600, and the Cube of 2 is 8; then say,

As 1600 : 8 :: 18225 : 91.125.

Then the Cube Root of 91.125 is 4.5, which is the Breadth; then 5 times 4.5 is 22.5, the Height, and 8 times 22.5 is 180 the Length.

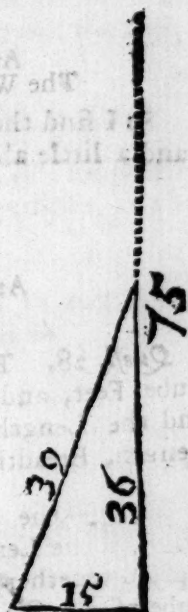
Quest. 29. There is a May-pole, whose Top-End was broken off by a Blast of Wind, and the Top-End in falling struck the Ground at 15 Feet Distance from the Foot of the May-pole, the broken Piece was 39 Feet; now I demand the Length of the May-pole?

By *Eucl. 1, 47*, the Square of the Hypotenuse of a right-angled Triangle, is equal to the Sum of the Squares of the Base and Perpendicular.

Therefore,

Therefore, from the Square of 39 subtract the Square of 15, the square Root of the Remainder is the Piece standing; to which add the Piece broken off, and you have the whole Length.

$$\begin{array}{r}
 39 \\
 39 \\
 \hline
 351 \\
 117 \\
 \hline
 1521 \\
 225 \\
 \hline
 1296(36 \\
 9 \\
 \hline
 66)396 \\
 396 \\
 \hline
 \dots
 \end{array}$$



The Piece standing is 36 Feet.

The Piece broken off is 39 Feet.

The whole Length 75 Feet.

Question 30.

A May-pole there was, whose Height I would know;
 The Sun shining clear, strait to work I did go:
 The Length of the Shadow, upon level Ground,
 Just sixty five Feet, when measur'd I found:
 A Staff I had there, just five Feet in Length;
 The Length of its Shadow was four Feet one Tenth:
 How high was the May-pole, I gladly would know?
 And it is the Thing you're desir'd to show.

By

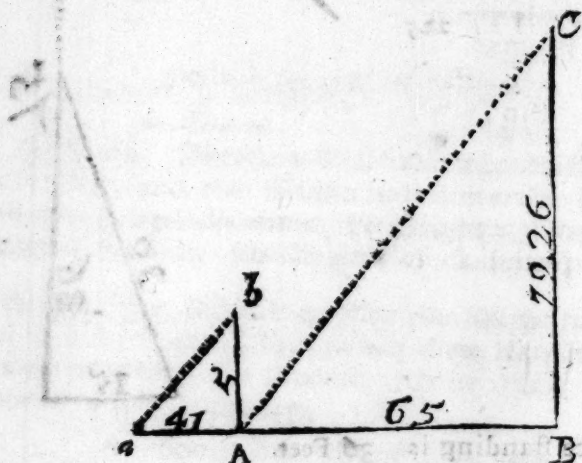
By *Euch.* 6. 4.

$$\text{As } aA : Ab :: AB : BC.$$

That is,

$$\text{As } 4.1 : 5 :: 65 : 79.26,$$

So I find the Height of the May-pole to be 79 Feet and a little above three Inches.



Here AB represents the Length of the Shadow of the May-pole, and BC the May-pole; *aA* the Shadow of the Staff, and *Ab* the Staff.

Quest. 31. What will be the Diameter of a Globe, when the Solidity and superficial Content thereof are equal?

If the Diameter be 1, the Solidity will be .5236, and the Superficies will be 3.1416; that is, as 1 to 6. And to find the superficial Content, we must multiply 3.1416 by the Square of the Axis, or Diameter, and the Product is the superficial Content. And for the Solidity, multiply .5236 by the Cube of the Axis, the Pro-

Product is the solid Content: Therefore, because .5236 is a sixth Part of 3.1416, we must take 6 for the Diameter sought. For if 3.1416 be multiply'd by the Square of 6, viz. by 36, the Product will be 113.0976; and if .5236 be multiply'd by the Cube of 6, viz. by 216, the Product is likewise 113.0976, the Solidity equal to the Superficies.

Therefore 6 is the true Answer.

Quest. 32. What will the Axis of a Globe be, when the Solidity is in Proportion to the Superficies, as 18 to 8?

Because the Solidity and Superficies is as 1 to 6, when the Axis of the Globe is 1, it will be

$$\text{As } 8 : 18 :: 6 : 13\frac{1}{2};$$

So the Diameter sought is $13\frac{1}{2}$.

If the Proportion of the Solidity to the Superficies had been as 8 to 18, then it will be

$$\text{As } 18 : 8 :: 6 : 2\frac{2}{3};$$

So then the Diameter will be $2\frac{2}{3}$.

The Reason of these Operations, both in this and the last Question, is from Algebra.

Quest. 33. There are three Grenado-Shells, of such Capacity, that the second Shell will just lye in the Concavity of the first, and the third in the Concavity of the second. The Solidity of the Metal of the first Shell is equal to its Concavity; and the Solidity of the Metal of the second, to the Concavity, is as 7 to 5; and the Solidity of the third, or least Shell's Metal, to its Concavity, is as 9 to 4. Now, supposing the Diameter of the first or greatest Shell, to be 16 In-

D d

ches

ches, and allowing every solid Inch of Iron to weigh 4 Ounces, I demand the Diameter of the two lesser Shells, and the Thickness and Solidity of Metal of every Shell, and also the Weight of every Shell?

The Cube of 16 is 4096; then,

As 1 : .5236 :: 4096 : 2144.6656;

the half thereof is 1072.3328, which is the Solidity of the Metal of the greater Shell, as also of the Concavity.

As .5236 : 1 :: 1072.3328 : 2048.

The Cube Root of 2048 is 12.699, which is the Diameter of the second Shell.

The Sum of 7 and 5 is 12; then,

As 12 : 5 :: 1072.3328 : 446.805.

This 446.805 is the solid Content of the Concavity of the second.

As .5236 : 1 :: 446.805 : 853.333.

The Cube Root of 853.333 is 9.485, the Diameter of the least Shell.

The Sum of 9 and 4 is 13; then,

As 13 : 4 :: 446.805 : 137.47846.

This 137.47846 is the solid Content of the Concavity of the third.

As .5236 : 1 :: 137.47846 : 262.5639.

The Cube Root of 262.5639 is 6.4034, the Diameter of the least Shell's Concavity.

From 16 the Diameter of the greatest,
subtr. 12.699 the Diameter of the second.

Rem. 3.301

half is = 1.65 the Thickness of Metal of the greatest.
From

From 12.699 the Diameter of the second,
 subtr. 9.485 the Diameter of the least.

Rem. 3.214

half is = 1.607 the Thickness of Metal of the second.

From 9.485 the Diameter of the least,
 subtract 6.403 the Diameter of the Concavity.

Remains 3.082

half is = 1.541 the Thickness of Metal of the least.

The Metal of the greatest is 1072.33 solid Inches ;
 which divide by 4, (because every solid Inch is a
 Quarter of a Pound) the Quotient is 268.08 Pounds.

The Metal of the second is 625.52 solid Inches ;
 which divided by 4, the Quotient is 156.38 Pounds,
 the Weight of the second.

The Metal of the least Shell is 309.32 solid Inches ;
 which divided by 4, the Quotient is 77.33 Pounds,
 the Weight of the least.

The Diam. { second Shell 12.699 } Inches.
 of the { least Shell — 9.485 }

The Thickness { greatest 1.65 } Inches.
 of the Metal { second 1.607 }
 of the { least — 1.541 }

The Weight { greatest 268.08 } Pounds.
 of the { second 156.38 }
 { least — 77.33 }

F I N I S.

ERRATA.

Page. Line.

- 18 8 for 6×6 , read 6×7
 30 14 for to, read will
 74 17 for Sides, read Side
 81 37 for FGKH, read FCKH
 99 { Prob. 7. The Figures are wrong plac'd in the
 Multiplication.
 101 8 for 1757, read 15757
 107 11 for 1041455, read 1001455
 121 17 for 26.64, read 20.64
 ditto 24 dele subtracted
 124 { 5 } for $\square Nn$, read $TS \times Nn$
 { 9 }
 125 29 read the Ellipsis and Circle are
 129 8 read to a Square
 180 18 for Semicircles, read Semicircle
 183 27 for ABC, read ABD
 185 1 for $\frac{Sr}{2}$ read $\frac{fr}{2}$
 189 11 for 1.5704, read 1.5708
 193 ult. dele (Eucl. 5.12)
 194 { 5,6 } for 3.8127, read 3.8197
 { 10 }
 ditto 17 for straiter, read straighter
 201 3 for $\square gn$, read $\square ne$
 216 18 dele (a)
 224 ult. for 10, read 1
 227 5 for 5.34, read 53.4
 237 penult. for superficial, read superfluous
 260 22 for 8145.792, read 814.5792
 280 20 dele and
 286 1 for 9 Inches, read 6 Inches
 298 20 for 4, read 4.5